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Premiums and reserves, adjusted by distortions. (English) Zbl 1398.91352

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Summary: The net premium principle is considered to be the most genuine and fair premium principle in actuarial applications. However, actuarial due diligence requires additional caution in pricing of insurance contracts to avoid, for example, at least bankruptcy of the insurer. This paper addresses the distorted premium principle from various angles. Distorted premiums are typically computed by underweighting or ignoring low, but overweighting high losses. Dual characterizations, which are elaborated in a first part of the paper, support this interpretation. The main contribution consists in an opposite point of view – an alternative characterization – which leaves the probability measure unchanged, but modifies (increases) the outcomes instead in a consistent way. It turns out that this new point of view is natural in actuarial practice, as it can be used for premium calculations, but equally well to determine the reserve process in subsequent years in a time consistent way.

MSC:

91B30 Risk theory, insurance (MSC2010)
60E15 Inequalities; stochastic orderings
62P05 Applications of statistics to actuarial sciences and financial mathematics

Keywords:

premium principles; dual representation; Fenchel-Young inequality; stochastic dominance

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