Ziesche, Sebastian

Sharpness of the phase transition and lower bounds for the critical intensity in continuum percolation on $\mathbb{R}^d$. (English. French summary) Zbl 1391.60246

Ann. Inst. Henri Poincaré, Probab. Stat. 54, No. 2, 866-878 (2018).

Summary: We consider the Boolean model Z on $\mathbb{R}^d$ with random compact grains of bounded diameter, i.e. $Z := \bigcup_{i\in\mathbb{N}}(Z_i + X_i)$ where $\{X_1, X_2, \ldots\}$ is a Poisson point process of intensity t and $(Z_1, Z_2, \ldots)$ is an i.i.d. sequence of compact grains (not necessarily balls) with diameters a.s. bounded by some constant. We will show that exponential decay holds in the sub-critical regime, that means the volume and radius of the cluster of the typical grain in Z have an exponential tail. To achieve this we adapt the arguments of *H. Duminil-Copin* and *V. Tassion* ["A new proof of the sharpness of the phase transition for Bernoulli percolation on $\mathbb{Z}^d$", Preprint, arXiv:1502.03051] and apply a new construction of the cluster of the typical grain together with arguments related to branching processes.

In the second part of the paper, we obtain new lower bounds for the Boolean model with deterministic grains. Some of these bounds are rigorous, while others are obtained via simulation. The simulated bounds come with confidence intervals and are much more precise than the rigorous ones. They improve known results [*S. Torquato* and *Y. Jiao*, "Effect of dimensionality on the continuum percolation of overlapping hyperspheres and hypercubes. II: Simulation results and analyses", J. Chem. Phys. 137, No. 7, Article ID 074106, 24 p. (2012; doi:10.1063/1.4742750)] in dimension six and above.

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
60D05 Geometric probability and stochastic geometry
60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)

Keywords:

Boolean model; Gilbert graph; Poisson process; exponential decay; continuum percolation; lower bound; critical intensity

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