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Connection formulas for general discrete Sobolev polynomials: Mehler-Heine asymptotics.
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Summary: In this paper the discrete Sobolev inner product

$$\langle p, q \rangle = \int p(x)q(x)d\mu + \sum_{i=0}^r M_i p^{(i)}(c)q^{(i)}(c)$$

is considered, where μ is a finite positive Borel measure supported on an infinite subset of the real line, $c \in \mathbb{R}$ and $M_i > 0$, $i = 0, 1, \dots, r$. Connection formulas for the orthonormal polynomials associated with $\langle \cdot, \cdot \rangle$ are obtained. As a consequence, for a wide class of measures μ , we give the Mehler-Heine asymptotics in the case of the point c is a hard edge of the support of μ . In particular, the case of a symmetric measure μ is analyzed. Finally, some examples are presented.

MSC:

- 42C05** Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Cited in 4 Documents

Keywords:

discrete Sobolev polynomials; connection formulas; Mehler-Heine type formulas; Bessel functions; asymptotic zero distribution

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