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The 2D Boussinesq equations with fractional horizontal dissipation and thermal diffusion.

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Summary: This paper examines the global regularity problem on the two-dimensional (2D) incompressible Boussinesq equations with fractional horizontal dissipation and thermal diffusion. The goal is to establish the global existence and regularity for the Boussinesq equations with minimal dissipation and thermal diffusion. By working with this general 1D fractional Laplacian dissipation, we are no longer constrained to the standard partial dissipation and this study will help understand the issue on how much dissipation is necessary for the global regularity. Due to the nonlocality of these 1D fractional operators, some of the standard energy estimate techniques such as integration by parts no longer apply and new tools including several anisotropic embedding and interpolation inequalities involving fractional derivatives are derived. These tools allow us to obtain very sharp upper bounds for the nonlinearities.

MSC:

35Q35	PDEs in connection with fluid mechanics
35B65	Smoothness and regularity of solutions to PDEs
76D03	Existence, uniqueness, and regularity theory for incompressible viscous fluids
35B45	A priori estimates in context of PDEs
35R11	Fractional partial differential equations

Keywords:

2D Boussinesq equations; fractional dissipation; global regularity

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