Towers, John D.

A source term method for Poisson problems on irregular domains. (English) Zbl 1422.65327

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Summary: This paper presents a finite difference method for solving Poisson problems on a two-dimensional irregular domain. An implicit, level set representation of the domain boundary is assumed, as well as a Cartesian grid that is not fitted to the domain. The algorithm is based on a scheme for interface problems which captures the jump conditions via singular source terms. This paper adapts that method to deal with boundary value problems by employing a simple iterative process that simultaneously enforces the boundary condition and solves for an unknown jump condition. The benefit and novelty of this method is that the boundary condition is captured via easily implemented source terms. The system of equations that results at each iteration can be solved using a FFT-based fast Poisson solver. The scheme can accommodate Dirichlet, Neumann, and Robin boundary conditions. We first address the constant coefficient Poisson equation, and then extend the scheme to accommodate the variable coefficient equation. Numerical examples indicate second order accuracy (or close to it) for the solution. The method also produces useful gradient approximations, but with generally lower convergence rates.

MSC:

65N06 Finite difference methods for boundary value problems involving PDEs
35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation

Keywords:

Poisson problem; irregular domain; level set methods; finite difference; Heaviside function; delta function

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