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Defective 2-colorings of planar graphs without 4-cycles and 5-cycles. (English) Zbl 1388.05072

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Summary: A 2-coloring is a coloring of vertices of a graph with colors 1 and 2. Define $V_i := \{v \in V(G) : c(v) = i\}$ for $i = 1$ and 2. We say that G is (d_1, d_2)-colorable if G has a 2-coloring such that V_i is an empty set or the induced subgraph $G[V_i]$ has the maximum degree at most d_i for $i = 1$ and 2. Let G be a planar graph without 4-cycles and 5-cycles. We show that the problem to determine whether G is $(0, k)$-colorable is NP-complete for every positive integer k. Moreover, we construct non-$(1, k)$-colorable planar graphs without 4-cycles and 5-cycles for every positive integer k. In contrast, we prove that G is (d_1, d_2)-colorable where $(d_1, d_2) = (4, 4), (3, 5)$, and $(2, 9)$.

MSC:

05C15 Coloring of graphs and hypergraphs
05C10 Planar graphs; geometric and topological aspects of graph theory
05C38 Paths and cycles
68Q25 Analysis of algorithms and problem complexity

Keywords:

defective coloring; planar graph; cycle

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.