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**On the compound Poisson risk model with periodic capital injections.** (English)

Zbl 1390.91220

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Summary: The analysis of capital injection strategy in the literature of insurance risk models typically assumes that whenever the surplus becomes negative, the amount of shortfall is injected so that the company can continue its business forever. Recently, *C. Nie* et al. [“Minimizing the ruin probability through capital injections”, Ann. Actuar. Sci. 5, No. 2, 195–209 (2011; doi:10.1017/S1748499511000054)] has proposed an alternative model in which capital is immediately injected to restore the surplus level to a positive level  $b$  when the surplus falls between zero and  $b$ , and the insurer is still subject to a positive ruin probability. Inspired by the idea of randomized observations in [*H. Albrecher* et al., Astin Bull. 41, No. 2, 645–672 (2011; Zbl 1239.91072)], in this paper, we further generalize Nie et al.’s [loc. cit.] model by assuming that capital injections are only allowed at a sequence of time points with inter-capital-injection times being Erlang distributed (so that deterministic time intervals can be approximated using the Erlangization technique in [*S. Asmussen* et al., Astin Bull. 32, No. 2, 267–281 (2002; Zbl 1081.60028)]). When the claim amount is distributed as a combination of exponentials, explicit formulas for the Gerber-Shiu expected discounted penalty function and the expected total discounted cost of capital injections before ruin are obtained. The derivations rely on a resolvent density associated with an Erlang random variable, which is shown to admit an explicit expression that is of independent interest as well. We shall provide numerical examples, including an application in pricing a perpetual reinsurance contract that makes the capital injections and demonstration of how to minimize the ruin probability via reinsurance. Minimization of the expected discounted capital injections plus a penalty applied at ruin with respect to the frequency of injections and the critical level  $b$  will also be illustrated numerically.

**MSC:**

91B30 Risk theory, insurance (MSC2010)

60K10 Applications of renewal theory (reliability, demand theory, etc.)

62P05 Applications of statistics to actuarial sciences and financial mathematics

Cited in 4 Documents

**Keywords:**

compound Poisson risk model; periodic capital injections; Gerber-Shiu expected discounted penalty function; resolvent measure; perpetual reinsurance

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**References:**

- [1] Albrecher, H.; Bäuerle, N.; Thonhauser, S., Optimal dividend-payout in random discrete time, *Statistics and Risk Modeling*, 28, 251-276, (2011) · Zbl 1233.91139
- [2] Albrecher, H.; Cheung, E. C.K.; Thonhauser, S., Randomized observation periods for the compound Poisson risk model: Dividends, *ASTIN Bulletin*, 41, 645-672, (2011) · Zbl 1239.91072
- [3] Albrecher, H.; Cheung, E. C.K.; Thonhauser, S., Randomized observation periods for the compound Poisson risk model: The discounted penalty function, *Scandinavian Actuarial Journal*, 2013, 424-452, (2013) · Zbl 1401.91089
- [4] Albrecher, H.; Ivanovs, J., A risk model with an observer in a Markov environment, *Risks*, 1, 148-161, (2013)
- [5] Albrecher, H.; Ivanovs, J., Strikingly simple identities relating exit problems for Lévy processes under continuous and Poisson observations, *Stochastic Processes and their Applications*, 127, 643-656, (2017) · Zbl 1354.60048
- [6] Albrecher, H.; Ivanovs, J.; Zhou, X., Exit identities for Lévy processes observed at Poisson arrival times, *Bernoulli*, 22, 1364-1382, (2016) · Zbl 1338.60125
- [7] Asmussen, S.; Avram, F.; Usabel, M., Erlangian approximations for finite-horizon ruin probabilities, *ASTIN Bulletin*, 32, 267-281, (2002) · Zbl 1081.60028
- [8] Avanzi, B.; Cheung, E. C.K.; Wong, B.; Woo, J.-K., On a periodic dividend barrier strategy in the dual model with continuous monitoring of solvency, *Insurance: Mathematics and Economics*, 52, 98-113, (2013) · Zbl 1291.91088

- [9] Avanzi, B.; Tu, V.; Wong, B., On optimal periodic dividend strategies in the dual model with diffusion., *Insurance: Mathematics and Economics*, 55, 210-224, (2014) · [Zbl 1296.91143](#)
- [10] Boxma, O. J.; Jonsson, H.; Resing, J. A.C.; Shneer, S., An alternating risk reserve process - Part II, *Markov Processes And Related Fields*, 16, 425-446, (2010) · [Zbl 1198.91085](#)
- [11] Cheung, E. C.K., Discussion of 'A direct approach to the discounted penalty function', *North American Actuarial Journal*, 14, 441-445, (2010)
- [12] Cheung, E. C.K., A unifying approach to the analysis of business with random gains, *Scandinavian Actuarial Journal*, 2012, 153-182, (2012) · [Zbl 1277.60148](#)
- [13] Choi, M. C.H.; Cheung, E. C.K., On the expected discounted dividends in the Cramér-Lundberg risk model with more frequent ruin monitoring than dividend decisions., *Insurance: Mathematics and Economics*, 59, 121-132, (2014) · [Zbl 1306.91072](#)
- [14] Dickson, D. C.M.; Qazvini, M., Gerber-Shiu analysis of a risk model with capital injections, *European Actuarial Journal*, 6, 409-440, (2016) · [Zbl 1394.91209](#)
- [15] Dickson, D. C.M.; Waters, H. R., Some optimal dividends problems, *ASTIN Bulletin*, 34, 49-74, (2004) · [Zbl 1097.91040](#)
- [16] Dufresne, D., Fitting combinations of exponentials to probability distributions, *Applied Stochastic Models in Business and Industry*, 23, 23-48, (2007) · [Zbl 1142.60321](#)
- [17] Eisenberg, J.; Schmidli, H., Minimising expected discounted capital injections by reinsurance in a classical risk model, *Scandinavian Actuarial Journal*, 2011, 155-176, (2011) · [Zbl 1277.60145](#)
- [18] Gerber, H. U.; Shiu, E. S.W., On the time value of ruin, *North American Actuarial Journal*, 2, 48-72, (1998)
- [19] Kulenko, N.; Schmidli, H., Optimal dividend strategies in a Cramér-Lundberg model with capital injections, *Insurance: Mathematics and Economics*, 43, 270-278, (2008) · [Zbl 1189.91075](#)
- [20] Kyprianou, A. E., *Gerber-Shiu Risk Theory*, (2013), Cham, Heidelberg, New York, Dordrecht, London: Springer, Cham, Heidelberg, New York, Dordrecht, London
- [21] Landriault, D.; Willmot, G. E., On the Gerber-Shiu discounted penalty function in the Sparre Andersen model with an arbitrary interclaim time distribution, *Insurance: Mathematics and Economics*, 42, 600-608, (2008) · [Zbl 1152.91591](#)
- [22] Liu, L.; Cheung, E. C.K., On a Gerber-Shiu type function and its applications in a dual semi-Markovian risk model., *Applied Mathematics and Computation*, 247, 1183-1201, (2014) · [Zbl 1338.60219](#)
- [23] Nie, C.; Dickson, D. C.M.; Li, S., Minimizing the ruin probability through capital injections, *Annals of Actuarial Science*, 5, 195-209, (2011)
- [24] Nie, C.; Dickson, D. C.M.; Li, S., The finite time ruin probability in a risk model with capital injections, *Scandinavian Actuarial Journal*, 2015, 301-318, (2015) · [Zbl 1398.91350](#)
- [25] Pafumi, G., Discussion of 'On the time value of ruin.', *North American Actuarial Journal*, 2, 75-76, (1998) · [Zbl 1081.91537](#)
- [26] Ramaswami, V.; Woolford, D. G.; Stanford, D. A., The Erlangization method for Markovian fluid flows, *Annals of Operations Research*, 160, 215-225, (2008) · [Zbl 1140.60357](#)
- [27] Stanford, D. A.; Avram, F.; Badescu, A. L.; Breuer, L.; Da Silva Soares, A.; Latouche, G., Phase-type approximations to finite-time ruin probabilities in the Sparre-Anderson and stationary renewal risk models, *ASTIN Bulletin*, 35, 131-144, (2005) · [Zbl 1123.62078](#)
- [28] Stanford, D. A.; Yu, K.; Ren, J., Erlangian approximation to finite time ruin probabilities in perturbed risk models, *Scandinavian Actuarial Journal*, 2011, 38-58, (2011) · [Zbl 1277.60128](#)
- [29] Zhang, Z., On a risk model with randomized dividend-decision times, *Journal of Industrial and Management Optimization*, 10, 1041-1058, (2014) · [Zbl 1282.91164](#)
- [30] Zhang, Z.; Cheung, E. C.K., The Markov additive risk process under an Erlangized dividend barrier strategy, *Methodology and Computing in Applied Probability*, 18, 275-306, (2016) · [Zbl 1338.91081](#)
- [31] Zhang, Z.; Cheung, E. C.K.; Yang, H., Lévy insurance risk process with Poissonian taxation, *Scandinavian Actuarial Journal*, 2017, 51-87, (2017) · [Zbl 1401.91216](#)

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