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A new computation of the critical point for the planar random-cluster model with $q \geq 1$.
(English. French summary) [Zbl 1395.82043](#)
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Authors' abstract: We present a new computation of the critical value of the random-cluster model with cluster weight $q \geq 1$ on \mathbb{Z}^2 . This provides an alternative approach to the result in [V. Beffara and H. Duminil-Copin, *Probab. Theory Relat. Fields* 153, No. 3–4, 511–542 (2012; [Zbl 1257.82014](#))]. We believe that this approach has several advantages. First, most of the proof can easily be extended to other planar graphs with sufficient symmetries. Furthermore, it invokes RSW-type arguments which are not based on self-duality. And finally, it contains a new way of applying sharp threshold results which avoid the use of symmetric events and periodic boundary conditions. Some of the new methods presented in this paper have a larger scope than the planar random-cluster model, and may be useful to investigate sharp threshold phenomena for more general dependent percolation processes in arbitrary dimensions.

Reviewer: [E. Ahmed \(Mansoura\)](#)

MSC:

- [82B20](#) Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
- [60K35](#) Interacting random processes; statistical mechanics type models; percolation theory
- [82B26](#) Phase transitions (general) in equilibrium statistical mechanics
- [82B43](#) Percolation
- [82B27](#) Critical phenomena in equilibrium statistical mechanics

Cited in **15** Documents

Keywords:

[phase transition](#); [random-cluster model](#); [Potts model](#); [critical point](#); [sharp phase transition](#)

Full Text: [DOI](#)

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