

Mironov, A.; Morozov, A.; Morozov, An.; Ramadevi, P.; Singh, Vivek Kumar
Colored HOMFLY polynomials of knots presented as double fat diagrams. (English)

Zbl 1388.57010

J. High Energy Phys. 2015, No. 7, Paper No. 109, 70 p. (2015).

Summary: Many knots and links in S^3 can be drawn as gluing of three manifolds with one or more four-punctured S^2 boundaries. We call these knot diagrams as double fat graphs whose invariants involve only the knowledge of the fusion and the braiding matrices of *four*-strand braids. Incorporating the properties of four-point conformal blocks in WZNW models, we conjecture colored HOMFLY polynomials for these double fat graphs where the color can be rectangular or non-rectangular representation. With the recent work of *J. Gu* and *H. Jockers* [Commun. Math. Phys. 338, No. 1, 393–456 (2015; Zbl 1328.81193)], the fusion matrices for the non-rectangular [21] representation, the first which involves multiplicity is known. We verify our conjecture by comparing with the [21] colored HOMFLY of many knots, obtained as closure of three braids. The conjectured form is computationally very effective leading to writing [21]-colored HOMFLY polynomials for many pretzel type knots and non-pretzel type knots. In particular, we find class of pretzel mutants which are distinguished and another class of mutants which cannot be distinguished by [21] representation. The difference between the [21]-colored HOMFLY of two mutants seems to have a general form, with A -dependence completely defined by the old conjecture due to Morton and Cromwell. In particular, we check it for an entire multi-parametric family of mutant knots evaluated using evolution method.

MSC:

- 57M27 Invariants of knots and 3-manifolds (MSC2010)
- 57R56 Topological quantum field theories (aspects of differential topology)
- 81T45 Topological field theories in quantum mechanics

Cited in 18 Documents

Keywords:

quantum groups; Chern-Simons theories; topological field theories

Software:

Knot Atlas

Full Text: DOI arXiv

References:

- [1] Alexander, JW, Topological invariants of knots and links, Trans. Amer. Math. Soc., 30, 275, (1928) · Zbl 54.0603.03
- [2] J.H. Conway, \textit{Algebraic properties}, in \textit{Computational problems in abstract algebra}, J. Leech ed., Proc. Conf. Oxford 1967, Pergamon Press, Oxford U.K. and New York U.S.A. (1970), pg. 329.
- [3] Jones, VFR, Index for subfactors, Invent. Math., 72, 1, (1983) · Zbl 0508.46040
- [4] Jones, VFR, A polynomial invariant for knots via von Neumann algebras, Bull. AMS, 12, 103, (1985) · Zbl 0564.57006
- [5] Jones, VFR, Hecke algebra representations of braid groups and link polynomials, Annals Math., 126, 335, (1987) · Zbl 0631.57005
- [6] Kauffman, L., State models and the Jones polynomial, Topology, 26, 395, (1987) · Zbl 0622.57004
- [7] Freyd, P.; et al., A new polynomial invariant of knots and links, Bull. AMS, 12, 239, (1985) · Zbl 0572.57002
- [8] Przytycki, JH; Traczyk, KP, Invariants of links of Conway type, Kobe J. Math., 4, 115, (1987) · Zbl 0655.57002
- [9] Chern, S-S; Simons, J., Characteristic forms and geometric invariants, Annals Math., 99, 48, (1974) · Zbl 0283.53036
- [10] Witten, E., Quantum field theory and the Jones polynomial, Commun. Math. Phys., 121, 351, (1989) · Zbl 0667.57005
- [11] N.M. Dunfield, S. Gukov and J. Rasmussen, \textit{The superpolynomial for knot homologies}, math/0505662 [INSPIRE]. · Zbl 1118.57012
- [12] Aganagic, M.; Shukirov, S., Knot homology and refined Chern-Simons index, Commun. Math. Phys., 333, 187, (2015) · Zbl 1322.81069

- [13] Dumin-Barkowski, P.; Mironov, A.; Morozov, A.; Sleptsov, A.; Smirnov, A., Superpolynomials for toric knots from evolution induced by cut-and-join operators, *JHEP*, 03, 021, (2013) · [Zbl 1342.57004](#)
- [14] I. Cherednik, \textit{Jones polynomials of torus knots via DAHA}, arXiv:1111.6195 [INSPIRE]. · [Zbl 1329.57019](#)
- [15] Gukov, S.; Stošić, M., Homological algebra of knots and BPS states, *Proc. Symp. Pure Math.*, 85, 125, (2012) · [Zbl 1296.57014](#)
- [16] S. Nawata, P. Ramadevi, Zodinmawia and X. Sun, \textit{Super-A-polynomials for twist knots}, \textit{JHEP}\textbf{11} (2012) 157 [arXiv:1209.1409] [INSPIRE]. · [Zbl 1333.81351](#)
- [17] S. Nawata, P. Ramadevi and Zodinmawia, \textit{Colored Kauffman homology and super-A-polynomials}, \textit{JHEP}\textbf{01} (2014) 126 [arXiv:1310.2240] [INSPIRE]. · [Zbl 1333.81351](#)
- [18] I.Cherednik and I.Danilenko, \textit{DAHA and iterated torus knots}, arXiv:1408.4348.
- [19] Khovanov, M., A categorification of the Jones polynomial, *Duke Math. J.*, 101, 359, (2000) · [Zbl 0960.57005](#)
- [20] Khovanov, M., Patterns in knot cohomology I, *Experiment. Math.*, 12, 365374, (2003) · [Zbl 1073.57007](#)
- [21] Khovanov, M., Categorifications of the colored Jones polynomial, *J. Knot Theor. Ramificat.*, 14, 111, (2005) · [Zbl 1083.57019](#)
- [22] Khovanov, M., $Sl(3)$ link homology, *Algebr. Geom. Topol.*, 4, 1045, (2004) · [Zbl 1159.57300](#)
- [23] Khovanov, M., Triply-graded link homology and Hochschild homology of Soergel bimodules, *Int. J. Math.*, 18, 869885, (2007) · [Zbl 1124.57003](#)
- [24] M. Khovanov, \textit{Link homology and categorification}, math/0605339. · [Zbl 1099.57008](#)
- [25] M. Khovanov, \textit{Categorifications from planar diagrammatics}, arXiv:1008.5084. · [Zbl 1226.81094](#)
- [26] Khovanov, M.; Rozhansky, L., Matrix factorizations and link homology, *Fund. Math.*, 199, 1, (2008) · [Zbl 1145.57009](#)
- [27] Khovanov, M.; Rozhansky, L., Matrix factorizations and link homology II, *Geom. Topol.*, 12, 1387, (2008) · [Zbl 1146.57018](#)
- [28] Bar-Natan, D., On khovanov's categorification of the Jones polynomial, *Alg. Geom. Topol.*, 2, 337, (2002) · [Zbl 0998.57016](#)
- [29] Bar-Natan, D., Khovanov's homology for tangles and cobordisms, *Geom. Topol.*, 9, 1443, (2005) · [Zbl 1084.57011](#)
- [30] Bar-Natan, D., Fast Khovanov homology computations, *J. Knot Theor. Ramificat.*, 16, 243255, (2007) · [Zbl 1234.57013](#)
- [31] Carqueville, N.; Murfet, D., Computing Khovanov-Rozansky homology and defect fusion, *Topology*, 14, 489, (2014) · [Zbl 1326.57024](#)
- [32] Dolotin, V.; Morozov, A., Introduction to Khovanov homologies. I. unreduced Jones superpolynomial, *JHEP*, 01, 065, (2013) · [Zbl 1342.57001](#)
- [33] Dolotin, V.; Morozov, A., Introduction to Khovanov homologies. II. reduced Jones superpolynomials, *J. Phys. Conf. Ser.*, 411, 012013, (2013)
- [34] Dolotin, V.; Morozov, A., Introduction to Khovanov homologies. III. A new and simple tensor-algebra construction of Khovanov-Rozansky invariants, *Nucl. Phys.*, B 878, 12, (2014) · [Zbl 1284.81150](#)
- [35] Anokhina, A.; Morozov, A., Towards R-matrix construction of Khovanov-Rozansky polynomials. I. primary T -deformation of HOMFLY, *JHEP*, 07, 063, (2014) · [Zbl 1333.81302](#)
- [36] A. Morozov, \textit{Challenges of β -deformation}, \textit{Theor. Math. Phys.}\textbf{173} (2012) 1417 [\textit{Teor. Mat. Fiz.}\textbf{173} (2012) 104] [arXiv:1201.4595] [INSPIRE].
- [37] Anokhina, A.; Mironov, A.; Morozov, A.; Morozov, A., Knot polynomials in the first non-symmetric representation, *Nucl. Phys.*, B 882, 171, (2014) · [Zbl 1285.81035](#)
- [38] E. Gorsky, S. Gukov and M. Stosic, \textit{Quadruply-graded colored homology of knots}, arXiv:1304.3481 [INSPIRE].
- [39] S. Arthamonov, A. Mironov and A. Morozov, \textit{Differential hierarchy and additional grading of knot polynomials}, \textit{Theor. Math. Phys.}\textbf{179} (2014) 509 [\textit{Teor. Mat. Fiz.}\textbf{179} (2014) 147] [arXiv:1306.5682] [INSPIRE]. · [Zbl 1333.57008](#)
- [40] A. Anokhina and An. Morozov, \textit{Cabling procedure for the colored HOMFLY polynomials}, \textit{Theor. Math. Phys.}\textbf{178} (2014) 1 [\textit{Teor. Mat. Fiz.}\textbf{178} (2014) 3] [arXiv:1307.2216] [INSPIRE]. · [Zbl 1318.81055](#)
- [41] Gu, J.; Jockers, H., A note on colored HOMFLY polynomials for hyperbolic knots from WZW models, *Commun. Math. Phys.*, 338, 393, (2015) · [Zbl 1328.81193](#)
- [42] Ramadevi, P.; Govindarajan, TR; Kaul, RK, Chirality of knots $9_{-}\{42\}$ and $10_{-}\{71\}$ and Chern-Simons theory, *Mod. Phys. Lett.*, A 9, 3205, (1994) · [Zbl 1015.57500](#)
- [43] S. Nawata, P. Ramadevi and Zodinmawia, \textit{Colored HOMFLY polynomials from Chern-Simons theory}, \textit{J. Knot Theor. Ramificat.}\textbf{22} (2013) 1350078 [arXiv:1302.5144] [INSPIRE]. · [Zbl 1296.57015](#)
- [44] Zodinmawia, \textit{Knot polynomials from} $SU(N)$ \textit{Chern-Simons theory, superpolynomials and super-A-polynomials}, Ph.D. thesis, IIT, Mumbai India (2014).
- [45] Galakhov, D.; Melnikov, D.; Mironov, A.; Morozov, A.; Sleptsov, A., Colored knot polynomials for arbitrary pretzel knots and links, *Phys. Lett.*, B 743, 71, (2015) · [Zbl 1343.57007](#)
- [46] A. Mironov, A. Morozov and A. Sleptsov, \textit{Colored HOMFLY polynomials for the pretzel knots and links}, arXiv:1412.8432 [INSPIRE].
- [47] D. Galakhov, D. Melnikov, A. Mironov and A. Morozov, \textit{Knot invariants from Virasoro related representation and pretzel knots}, arXiv:1502.02621 [INSPIRE]. · [Zbl 1331.81256](#)
- [48] D. Bar-Natan, \textit{The Knot atlas webpage}, <http://www.katlas.org>.

- [49] A. Kawauchi, *Survey on knot theory*, Springer, Germany (1996).
- [50] Kaul, RK; Govindarajan, TR, Three-dimensional Chern-Simons theory as a theory of knots and links, Nucl. Phys., B 380, 293, (1992) · [Zbl 0938.81553](#)
- [51] Kaul, RK; Govindarajan, TR, Three-dimensional Chern-Simons theory as a theory of knots and links. 2. multicolored links, Nucl. Phys., B 393, 392, (1993) · [Zbl 1245.57011](#)
- [52] Rama Devi, P.; Govindarajan, TR; Kaul, RK, Three-dimensional Chern-Simons theory as a theory of knots and links. 3. compact semisimple group, Nucl. Phys., B 402, 548, (1993) · [Zbl 0941.57500](#)
- [53] Ramadevi, P.; Govindarajan, TR; Kaul, RK, Knot invariants from rational conformal field theories, Nucl. Phys., B 422, 291, (1994) · [Zbl 0990.81694](#)
- [54] Ramadevi, P.; Govindarajan, TR; Kaul, RK, Representations of composite braids and invariants for mutant knots and links in Chern-Simons field theories, Mod. Phys. Lett., A 10, 1635, (1995) · [Zbl 1020.57510](#)
- [55] Ramadevi, P.; Sarkar, T., On link invariants and topological string amplitudes, Nucl. Phys., B 600, 487, (2001) · [Zbl 1097.81742](#)
- [56] Zodinmawia and P. Ramadevi, *SU(N) quantum Racah coefficients & non-torus links*, *Nucl. Phys.* **B 870** (2013) 205 [arXiv:1107.3918] [INSPIRE]. · [Zbl 1262.81168](#)
- [57] Zodinmawia and P. Ramadevi, *Reformulated invariants for non-torus knots and links*, arXiv:1209.1346 [INSPIRE]. · [Zbl 1262.81168](#)
- [58] N. Yu. Reshetikhin and V.G. Turaev, *Ribbon graphs and their invariants derived from quantum groups*, *Commun. Math. Phys.* **127** (1990) 1 [INSPIRE]. · [Zbl 0768.57003](#)
- [59] E. Guadagnini, M. Martellini and M. Mintchev, *Chern-Simons field theory and quantum groups*, in *Proceedings of the Workshop on Quantum Groups, Clausthal* 1989, World Scientific, Singapore (1989), pg. 307 [*Lect. Notes Phys.*] **370** (1990) 307 [INSPIRE]. · [Zbl 0722.57003](#)
- [60] Guadagnini, E.; Martellini, M.; Mintchev, M., Chern-Simons holonomies and the appearance of quantum groups, Phys. Lett., B 235, 275, (1990) · [Zbl 0722.57003](#)
- [61] Turaev, VG; Viro, OY, State sum invariants of 3 manifolds and quantum 6j symbols, *Topology*, 31, 865, (1992) · [Zbl 0779.57009](#)
- [62] Morozov, A.; Smirnov, A., Chern-Simons theory in the temporal gauge and knot invariants through the universal quantum R-matrix, Nucl. Phys., B 835, 284, (2010) · [Zbl 1204.81097](#)
- [63] A. Smirnov, *Notes on Chern-Simons theory in the temporal gauge*, in *Proc. of International School of Subnuclear Phys.*, Erice Italy (2009) [*Subnucl. Ser.*] **47** (2011) 489 [arXiv:0910.5011] [INSPIRE].
- [64] A. Mironov, A. Morozov and An. Morozov, *Character expansion for HOMFLY polynomials. II. Fundamental representation. Up to five strands in braid*, *JHEP* **03** (2012) 034 [arXiv:1112.2654] [INSPIRE]. · [Zbl 1309.81114](#)
- [65] A. Mironov, A. Morozov and A. Morozov, *Character expansion for HOMFLY polynomials. I. Integrability and difference equations*, in *Strings, Gauge Fields, and the Geometry Behind: The Legacy of Maximilian Kreuzer*, A. Rebhan, L. Katzarkov, J. Knapp, R. Rashkov and E. Scheidegger eds., World Scientific, Singapore (2013), pg. 101 [arXiv:1112.5754].
- [66] H. Itoyama, A. Mironov, A. Morozov and An. Morozov, *Character expansion for HOMFLY polynomials. III. All 3-strand braids in the first symmetric representation*, *Int. J. Mod. Phys.* **A 27** (2012) 1250099 [arXiv:1204.4785] [INSPIRE]. · [Zbl 1260.81134](#)
- [67] A. Anokhina, A. Mironov, A. Morozov and An. Morozov, *Racah coefficients and extended HOMFLY polynomials for all 5- and 7-strand braids*, *Nucl. Phys.* **B 868** (2013) 271 [arXiv:1207.0279] [INSPIRE]. · [Zbl 1262.81073](#)
- [68] Itoyama, H.; Mironov, A.; Morozov, A.; Morozov, A., Eigenvalue hypothesis for racah matrices and HOMFLY polynomials for 3-strand knots in any symmetric and antisymmetric representations, *Int. J. Mod. Phys.*, A 28, 1340009, (2013) · [Zbl 1259.81082](#)
- [69] A. Anokhina, A. Mironov, A. Morozov and An. Morozov, *Colored HOMFLY polynomials as multiple sums over paths or standard Young tableaux*, *Adv. High Energy Phys.* **2013** (2013) 931830 [arXiv:1304.1486] [INSPIRE].
- [70] Mironov, A.; Morozov, A.; Morozov, A., Evolution method and “differential hierarchy” of colored knot polynomials, *AIP Conf. Proc.*, 1562, 123, (2013)
- [71] Mironov, A.; Morozov, A.; Natanzon, S., Integrability properties of Hurwitz partition functions. II. multiplication of cut-and-join operators and WDVV equations, *JHEP*, 11, 097, (2011) · [Zbl 1306.81291](#)
- [72] Mironov, A.; Morozov, A.; Natanzon, S., Complete set of cut-and-join operators in Hurwitz-Kontsevich theory, *Theor. Math. Phys.*, 166, 1, (2011) · [Zbl 1312.81125](#)
- [73] Mironov, A.; Morozov, A.; Natanzon, S., Algebra of differential operators associated with Young diagrams, *J. Geom. Phys.*, 62, 148, (2012) · [Zbl 1242.22008](#)
- [74] Morton, H.; Cromwell, P., Distinguishing mutants by knot polynomials, *J. Knot Theor. Ramificat.*, 5, 225, (1996) · [Zbl 0866.57002](#)
- [75] J. Murakami, *Finite type invariants detecting the mutant knots*, in *Knot Theory, a volume dedicated to Professor Kunio Murasugi for his 70th birthday*, M. Sakuma et al. eds., Osaka University, Osaka Japan (2000), pg. 258.
- [76] Rosso, M.; Jones, VFR, On the invariants of torus knots derived from quantum groups, *J. Knot Theor. Ramificat.*, 2, 97, (1993) · [Zbl 0787.57006](#)
- [77] Lin, X-S; Zheng, H., On the Hecke algebras and the colored HOMFLY polynomial, *Trans. Amer. Math. Soc.*, 362, 1, (2010) · [Zbl 1193.57006](#)

- [78] A. Mironov, A. Morozov and An. Morozov, \textit{On colored HOMFLY polynomials for twist knots}, \textit{Mod. Phys. Lett.}\textbf{A 29} (2014) 1450183 [arXiv:1408.3076] [INSPIRE].
- [79] S. Nawata, P. Ramadevi and Zodinmawia, \textit{Multiplicity-free quantum } 6\textit{-symbols for } U\textit{-}\{\}\{\\$q\\$}\textit{ (}\textit{Lett. Math. Phys.}\textit{)\textbf{103} (2013) 1389 [arXiv:1302.5143] [INSPIRE].
- [80] H. Itoyama, A. Mironov, A. Morozov and An. Morozov, \textit{HOMFLY and superpolynomials for figure eight knot in all symmetric and antisymmetric representations}, \textit{JHEP}\textbf{07} (2012) 131 [arXiv:1203.5978] [INSPIRE].
- [81] Zhu, S., Colored HOMFLY polynomials via skein theory, JHEP, 10, 229, (2013)
- [82] Morozov, A., Special colored superpolynomials and their representation-dependence, JHEP, 12, 116, (2012)
- [83] Morozov, A., The first-order deviation of superpolynomial in an arbitrary representation from the special polynomial, JETP Lett., 97, 171, (2013)
- [84] A. Mironov, A. Morozov and A. Sleptsov, \textit{Genus expansion of HOMFLY polynomials}, \textit{Theor. Math. Phys.}\textbf{177} (2013) 1435 [\textit{Teor. Mat. Fiz.}\textbf{177} (2013) 179] [arXiv:1303.1015] [INSPIRE]. · [Zbl 1336.57022](#)
- [85] Mironov, A.; Morozov, A.; Sleptsov, A., On genus expansion of knot polynomials and hidden structure of Hurwitz tau-functions, Eur. Phys. J., C 73, 2492, (2013)
- [86] Mironov, A.; Morozov, A.; Sleptsov, A.; Smirnov, A., On genus expansion of superpolynomials, Nucl. Phys., B 889, 757, (2014) · [Zbl 1326.57030](#)
- [87] Stoimenow, A., Tabulating and distinguishing mutants, Int. J. Alg. Comput., 20, 525, (2010) · [Zbl 1195.57021](#)
- [88] A. Stoimenow, \textit{Knot data tables webpage}, <http://stoimenov.net/stoimeno/homepage/ptab/index.html>.
- [89] D. De Wit and J. Links, \textit{Where the Links-Gould invariant first fails to distinguish nonmutant prime knots}, math/0501224. · [Zbl 1187.57013](#)
- [90] S. Nawata, P. Ramadevi and V.K. Singh, \textit{Colored HOMFLY polynomials can distinguish mutant knots}, arXiv:1504.00364 [INSPIRE]. · [Zbl 1405.57017](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.