Peng, Junhao; **Agliari, Elena**
Scaling laws for diffusion on (trans)fractal scale-free networks. (English) Zbl 1390.28021
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Summary: Fractal (or transfractal) features are common in real-life networks and are known to influence the dynamic processes taking place in the network itself. Here, we consider a class of scale-free deterministic networks, called (u, v)-flowers, whose topological properties can be controlled by tuning the parameters u and v; in particular, for $u > 1$, they are fractals endowed with a fractal dimension df, while for $u = 1$, they are transfractal endowed with a transfractal dimension $\widetilde{d}_f$. In this work, we investigate dynamic processes (i.e., random walks) and topological properties (i.e., the Laplacian spectrum) and we show that, under proper conditions, the same scalings (ruled by the related dimensions) emerge for both fractal and transfractal dimensions.

MSC:

28A80 Fractals
90B10 Deterministic network models in operations research
05C81 Random walks on graphs

Full Text: DOI

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[56] Notice that the radius of convergence of a probability generating function must be at least 1, and, in particular, the normalizaton of \textit{p_{k}} yields to $\Phi_T(z) = 1$. Also, a diverging derivative as $z \rightarrow 1$ means that the first moment is diverging as well and similarly for higher order moments. For the quantities considered here (i.e., FPT, FRT, and GFPT), given the finiteness of the underling structure, the moments are all finite. We also recall that the probability generating function of \textit{p_{k}} can also be seen as the (discrete) Laplace transformation of \textit{p_{k}}. In this perspective, the discrete function to be transformed does not need to be normalized.

[57] Notice that the probability distribution for $T_{H_0 \rightarrow H_0}(t)$ is not normalized over time and, accordingly, $\Phi_{R T}(t, z)$ is diverging for $z \rightarrow 1$. Nonetheless, one can exploit $\Phi_{R T}(t, z)$ to obtain closed-form, well-defined expression for $\Phi_{F P T}(t, z), \Phi_{F R T}(t, z)$, and $\Phi_{\text{GFPT}}(t, z)$. See, e.g., Refs. 21 and 55 for further details.\par