Duminil-Copin, Hugo; **Gagnebin, Maxime**; **Harel, Matan**; **Manolescu, Ioan**; **Tassion, Vincent**
The Bethe ansatz for the six-vertex and XXZ models: an exposition. (English) Zbl 1430.60080
Probab. Surv. 15, 102-130 (2018).

Summary: In this paper, we review a few known facts on the coordinate Bethe ansatz. We present a detailed construction of the Bethe ansatz vector ψ and energy Λ, which satisfy $V\psi = \Lambda\psi$, where V is the transfer matrix of the six-vertex model on a finite square lattice with periodic boundary conditions for weights $a = b = 1$ and $c > 0$. We also show that the same vector ψ satisfies $H\psi = E\psi$, where H is the Hamiltonian of the XXZ model (which is the model for which the Bethe ansatz was first developed), with a value E computed explicitly.

Variants of this approach have become central techniques for the study of exactly solvable statistical mechanics models in both the physics and mathematics communities. Our aim in this paper is to provide a pedagogically-minded exposition of this construction, aimed at a mathematical audience. It also provides the opportunity to introduce the notation and framework which will be used in a subsequent paper by the authors ["Discontinuity of the phase transition for the planar random-cluster and Potts models with $q > 4$", Preprint, `arXiv:1611.09877`] that amounts to proving that the random-cluster model on $\mathbb{Z}^2$ with cluster weight $q > 4$ exhibits a first-order phase transition.

MSC:

60K35	Interacting random processes; statistical mechanics type models; percolation theory	Cited in **10** Documents
82B20	Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics	
82B23	Exactly solvable models; Bethe ansatz	
82B26	Phase transitions (general) in equilibrium statistical mechanics	

Keywords:

transfer matrix; Bethe ansatz; six vertex model; XXZ model

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