Shampine, L. F.; Zhang, W.

Efficient integration of ordinary differential equations by transformation. (English)

Zbl 0686.65035
Comput. Math. Appl. 15, No. 3, 213-220 (1988).

The paper deals with the numerical solution of initial value problems of ordinary differential equations with a parameter ϵ such as $y' = \epsilon f(x, y, \epsilon)$. Under certain assumptions this problem becomes easier for the typical numerical method as $\epsilon \to 0$, still the problem might be difficult for non-vanishing ϵ. However, if the limit problem with $\epsilon = 0$ may be solved analytically, it is possible to transform the original problem to a form which is easier for numerical methods. For practical purposes this transformation must be well-conditioned.

The paper gives as beautiful example for this method a stiff ordinary differential equation with slowly varying Jacobian and refers to similar techniques by *J. D. Lawson* [SIAM J. Numer. Anal. 4, 372-380 (1967; Zbl 0223.65030)] and *P. Deuflhard* [Z. Angew. Math. Phys. 30, 177- 189 (1979; Zbl 0406.70012); Celestial Mech. 21, 213-223 (1980; Zbl 0422.70019)]. The displacement of a nonlinear spring and Bessel's equation illustrate the numerical behaviour of this technique and the improvements on the computation time. Compared to other approaches one of the main advantages of this method is the possibility to transform the equation and to use existing codes, it is not necessary to develop new software, whereas the main problems are the analytic solution and the condition of the transformation.

Reviewer: C.H.Cap

MSC:

65L05 Numerical methods for initial value problems involving ordinary differential equations

34A34 Nonlinear ordinary differential equations and systems

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stiff system; parameter; transformation; slowly varying Jacobian; nonlinear spring; Bessel's equation

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