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Set-valued dynamics related to generalized Euler-Lagrange functional equations. (English)

Zbl 1396.39020
J. Fixed Point Theory Appl. 20, No. 1, Paper No. 32, 8 p. (2018).

Suppose that A is a convex cone in a real vector space X with $0 \in A$ and let S be a set-valued mapping from $A - A$ (see the paper for the definition of the minus operation) to the family of all nonempty convex and closed subsets of a Banach space Y, containing the origin of Y. Using Minkowski addition and scalar multiplication, one can associate to every finite family $x_1, \dots, x_n$ of X the operators $\biguplus_{x_2} S(x_1) = S(x_1 + x_2) + S(x_1 - x_2)$ and $\biguplus_{x_2,\dots,x_n} S(x_1) = \biguplus_{x_n} \left(\biguplus_{x_2,\dots,x_{n-1}} S(x_1)\right)$ for $n \geq 3$.

The author provide an interesting characterization of the set-valued mappings S that verify (for some positive numbers $a_1 > 1, \dots, a_n = 1$) the cubic inclusion

$$\biguplus_{a_2x_2,\dots,a_nx_n} S(a_1x_1) + 2^{n-1}a_1 \sum_{i=2}^{n} a_i^2 S(x_i) \subseteq 2^{n-2}a_1 \sum_{i=2}^{n} a_i^2 \biguplus_{x_i} S(x_1) + 2^{n-1}a_1^3 S(x_1),$$

for all $x_1, \dots, x_n \in A$, and also the boundedness condition $\sup\{\operatorname{diam}(S(x)) : x \in A\} < \infty$. Precisely, the existence and uniqueness of a cubic mapping $C : A - A \to Y$ such that $C(x) \in S(x)$ for all $x \in A$ is proved. A quartic analogue of this result is also included.

Reviewer: Constantin Niculescu (Craiova)

MSC:

39B52	Functional equations for functions with more general domains and/or ranges
39B62	Functional inequalities, including subadditivity, convexity, etc.
54C60	Set-valued maps in general topology
47H04	Set-valued operators
52A07	Convex sets in topological vector spaces (aspects of convex geometry)
39B82	Stability, separation, extension, and related topics for functional equations

Keywords:

Euler-Lagrange functional equations; Minkowski operations; convex set; convex cone

Full Text: DOI

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