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Fibrations in CICY threefolds. (English) [Zbl 1383.83147](#)
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Summary: In this work we systematically enumerate genus one fibrations in the class of 7,890 Calabi-Yau manifolds defined as complete intersections in products of projective spaces, the so-called CICY threefolds. This survey is independent of the description of the manifolds and improves upon past approaches that probed only a particular algebraic form of the threefolds (i.e. searches for “obvious” genus one fibrations as in [*J. Gray et al.*, “Topological invariants and fibration structure of complete intersection Calabi-Yau four-folds”, Preprint, [arXiv:1405.2073](#)] and [*L. B. Anderson et al.*, “Multiple fibrations in Calabi-Yau geometry and string dualities”, Preprint, [arXiv:1608.07555](#)]). We also study K3-fibrations and nested fibration structures. That is, K3 fibrations with potentially many distinct elliptic fibrations. To accomplish this survey a number of new geometric tools are developed including a determination of the full topology of all CICY threefolds, including triple intersection numbers. In 2, 946 cases this involves finding a new “favorable” description of the manifold in which all divisors descend from a simple ambient space. Our results consist of a survey of obvious fibrations for all CICY threefolds and a complete classification of all genus one fibrations for 4,957 “Kähler favorable” CICYs whose Kähler cones descend from a simple ambient space. Within the CICY dataset, we find 139,597 obvious genus one fibrations, 30,974 obvious K3 fibrations and 208,987 nested combinations. For the Kähler favorable geometries we find a complete classification of 377,559 genus one fibrations. For one manifold with Hodge numbers (19, 19) we find an explicit description of an infinite number of distinct genus-one fibrations extending previous results for this particular geometry that have appeared in the literature. The data associated to this scan is available in [<http://www1.phys.vt.edu/cicydata/>].

MSC:

- 83E30 String and superstring theories in gravitational theory
- 14J32 Calabi-Yau manifolds (algebraic-geometric aspects)
- 32Q25 Calabi-Yau theory (complex-analytic aspects)
- 81T30 String and superstring theories; other extended objects (e.g., branes) in quantum field theory
- 53B35 Local differential geometry of Hermitian and Kählerian structures

Cited in **21** Documents

Keywords:

differential and algebraic geometry; F-theory; superstring vacua

Software:

CICY Quotients

Full Text: [DOI](#)

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