Tian, Yongge; **Jiang, Bo**

Quadratic properties of least-squares solutions of linear matrix equations with statistical applications. (English) Zbl 1417.15018

Comput. Stat. 32, No. 4, 1645-1663 (2017).

Summary: Assume that a quadratic matrix-valued function $\psi(X) = Q - X'PX$ is given and let $\mathcal{S} = \{X \in \mathbb{R}^{n\times m} \mid \text{trace}[(AX - B)'(AX - B)] = \min\}$ be the set of all least-squares solutions of the linear matrix equation $AX = B$. In this paper, we first establish explicit formulas for calculating the maximum and minimum ranks and inertias of $\psi(X)$ subject to $X \in \mathcal{S}$, and then derive from the formulas the analytic solutions of the two optimization problems $\psi(X) = \max$ and $\psi(X) = \min$ subject to $X \in \mathcal{S}$ in the Löwner partial ordering. As applications, we present a variety of results on equalities and inequalities of the ordinary least squares estimators of unknown parameter vectors in general linear models.

MSC:

15A24 Matrix equations and identities
15B57 Hermitian, skew-Hermitian, and related matrices
62J05 Linear regression; mixed models
62H12 Estimation in multivariate analysis

Cited in **1** Document

Keywords:

quadratic matrix-valued function; rank; inertia; Löwner partial ordering; linear model

Full Text: DOI

References:

[1] Cain, BE; Sá, EM, The inertia of Hermitian matrices with a prescribed 2×2 block decomposition, Linear Multilinear Algebra, 31, 119-130, (1992) · Zbl 0756.15024

[2] Chabrillac, Y; Crouzeix, JP, Definiteness and semidefiniteness of quadratic forms revisited, Linear Algebra Appl, 63, 283-292, (1984) · Zbl 0548.15027

[3] Cohen, N; Dancis, J, Inertias of block band matrix completions, SIAM J Matrix Anal Appl, 9, 583-612, (1999) · Zbl 0974.15009

[4] Fonseca, CM, The inertia of certain Hermitian block matrices, Linear Algebra Appl, 274, 193-210, (1998) · Zbl 0929.15019

[5] Dancis, J, The possible inertias for a Hermitian matrix and its principal submatrices, Linear Algebra Appl, 85, 121-151, (1987) · Zbl 0614.15011

[6] Dancis, J, Several consequences of an inertia theorem, Linear Algebra Appl, 136, 43-61, (1990) · Zbl 0712.15021

[7] Dong, B; Guo, W; Tian, Y, On relations between BLUEs under two transformed linear models, J Multivar Anal, 131, 279-292, (2014) · Zbl 1299.62055

[8] Graybill FA (1961) An introduction to linear statistical models, vol I. McGraw-Hill, New York · Zbl 0121.35605

[9] Hatcher A (2002) Algebraic topology. Cambridge University Press, Cambridge · Zbl 1044.55001

[10] Haynsworth, EV, Determination of the inertia of a partitioned Hermitian matrix, Linear Algebra Appl, 1, 73-81, (1968) · Zbl 0155.06304

[11] Haynsworth, EV; Ostrowski, AM, On the inertia of some classes of partitioned matrices, Linear Algebra Appl, 1, 299-316, (1968) · Zbl 0186.33704

[12] James IJ (1976) The topology of Stiefel manifolds. Cambridge University Press, Cambridge · Zbl 0337.55017

[13] Johnson, CR; Rodman, L, Inertia possibilities for completions of partial Hermitian matrices, Linear Multilinear Algebra, 16, 179-195, (1984) · Zbl 0548.15020

[14] Liu, Y; Tian, Y, More on extremal ranks of the matrix expressions $A-BX \pm X^{*}B^{*}$ with statistical applications, Numer Linear Algebra Appl, 15, 307-325, (2008) · Zbl 1212.15029

[15] Liu, Y; Tian, Y, MAX-MIN problems on the ranks and inertias of the matrix expressions $A-BXC \pm (BXC)^{*}$ with applications, J Optim Theory Appl, 148, 593-622, (2011) · Zbl 1223.90077

[16] Marsaglia, G; Styan, GPH, Equalities and inequalities for ranks of matrices, Linear Multilinear Algebra, 2, 269-292, (1974) · Zbl 0297.15003

[17] Penrose, R, A generalized inverse for matrices, Proc Camb Philos Soc, 51, 406-413, (1955) · Zbl 0065.24603

[18] Penrose, R, On best approximate solution of linear matrix equations, Proc Camb Philos Soc, 52, 17-19, (1956) · Zbl 0070.12501

[19] Searle SR (1971) Linear models. Wiley, New York · Zbl 0218.62071

[20] Tian, Y, Equalities and inequalities for inertias of Hermitian matrices with applications, Linear Algebra Appl, 433, 263-296, (2010) · Zbl 1205.15033

[21] Tian, Y, Maximization and minimization of the rank and inertia of the Hermitian matrix expression $A - BX - (BX)^{*}$ with applications, Linear Algebra Appl, 434, 2109-2139, (2011) · Zbl 1211.15022

[22] Tian, Y, Solving optimization problems on ranks and inertias of some constrained nonlinear matrix functions via an algebraic linearization method, Nonlinear Anal, 75, 717-734, (2012) · Zbl 1236.65070

[23] Tian, Y, Formulas for calculating the extremum ranks and inertias of a four-term quadratic matrix-valued function and their applications, Linear Algebra Appl, 437, 835-859, (2012) · Zbl 1252.15026

[24] Tian, Y, Solutions to 18 constrained optimization problems on the rank and inertia of the linear matrix function $A + BXB^{*}$, Math Comput Model, 55, 955-968, (2012) · Zbl 1255.15010

[25] Tian, Y, Some optimization problems on ranks and inertias of matrix-valued functions subject to linear matrix equation restrictions, Banach J Math Anal, 8, 148-178, (2014) · Zbl 1278.15019

[26] Tian, Y, A survey on rank and inertia optimization problems of the matrix-valued function $A + BXB^{*}$, Numer Algebra Contr Optim, 5, 289-326, (2015) · Zbl 1327.15007

[27] Tian, Y; Kyrchei, I (ed.), How to characterize properties of general Hermitian quadratic matrix-valued functions by rank and inertia, 150-183, (2015), New York

[28] Tian, Y, A new derivation of BLUPs under random-effects model, Metrika, 78, 905-918, (2015) · Zbl 1329.62264

[29] Tian, Y, A matrix handling of predictions under a general linear random-effects model with new observations, Electron J Linear Algebra, 29, 30-45, (2015) · Zbl 1329.62321

[30] Tian, Y; Jiang, B, Equalities for estimators of partial parameters under linear model with restrictions, J Multivar Anal, 143, 299-313, (2016) · Zbl 1328.62347

[31] Tian, Y; Zhang, X, On connections among OLSEs and BLUEs of whole and partial parameters under a general linear model, Stat Prob Lett, 112, 105-112, (2016) · Zbl 1341.62133

[32] Zhang, X; Tian, Y, On decompositions of BLUEs under a partitioned linear model with restrictions, Stat Pap, 57, 345-364, (2016) · Zbl 1341.62142