Mehlmann, C.; **Richter, T.**

A finite element multigrid-framework to solve the sea ice momentum equation. (English)

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Summary: We present a multigrid framework for the solution of the momentum equation arising in Hibler's viscous-plastic (VP) sea ice model. This model is used in global climate models, for seasonal prediction and high-resolution short-term forecasting systems. The development of fast and robust solvers for the strongly nonlinear momentum equation is still a big issue. There are mainly three approaches to solve the momentum equation: a fixed-point iteration (Picard solver), an inexact Newton method and a subcycling procedure based on an elastic-viscous-plastic (EVP) model approximation. Simple fixed-point iterations call for a vast number of cycles. The Jacobian arising in Newton linearizations is ill-conditioned and unstructured. No efficient linear solvers are available up to date. One possibility to solve the linear systems arising within a Newton method is the preconditioned GMRES iteration that however still requires many steps. The commonly used line SOR preconditioner is computationally expensive. Especially on fine meshes, literature recommends to revise common solution strategies. We introduce a geometric multigrid method as a preconditioner to the GMRES iteration for accelerating the solution of the linear problems. We show that the convergence rate of the multigrid method is robust with respect to mesh refinement. This makes it an appealing method for high resolution simulations. We validate the robustness of the linear solver and compare the multigrid with ILU preconditioning. In particular on fine meshes ($\sim 16\,\mathrm{km}$–$2\,\mathrm{km}$), multigrid preconditioning can substantially reduce the computational effort and decreases iteration counts by 80%.

MSC:

86A05 Hydrology, hydrography, oceanography
65M55 Multigrid methods; domain decomposition for initial value and initial-boundary value problems involving PDEs
65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs

Keywords:

sea ice dynamics; multigrid methods; finite elements

Software:

GASCOIGNE; NewtonLib; Wesseling

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