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Elementary dispersion analysis of some mimetic discretizations on triangular C-grids. (English) Zbl 1380.65220

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Summary: Spurious modes supported by triangular C-grids limit their application for modeling large-scale atmospheric and oceanic flows. Their behavior can be modified within a mimetic approach that generalizes the scalar product underlying the triangular C-grid discretization. The mimetic approach provides a discrete continuity equation which operates on an averaged combination of normal edge velocities instead of normal edge velocities proper. An elementary analysis of the wave dispersion of the new discretization for Poincaré, Rossby and Kelvin waves shows that, although spurious Poincaré modes are preserved, their frequency tends to zero in the limit of small wavenumbers, which removes the divergence noise in this limit. However, the frequencies of spurious and physical modes become close on shorter scales indicating that spurious modes can be excited unless high-frequency short-scale motions are effectively filtered in numerical codes. We argue that filtering by viscous dissipation is more efficient in the mimetic approach than in the standard C-grid discretization. Lumping of mass matrices appearing with the velocity time derivative in the mimetic discretization only slightly reduces the accuracy of the wave dispersion and can be used in practice. Thus, the mimetic approach cures some difficulties of the traditional triangular C-grid discretization but may still need appropriately tuned viscosity to filter small scales and high frequencies in solutions of full primitive equations when these are excited by nonlinear dynamics.

MSC:

65M22 Numerical solution of discretized equations for initial value and initial-boundary value problems involving PDEs
35Q35 PDEs in connection with fluid mechanics
35Q86 PDEs in connection with geophysics

Keywords:

C-grid; dispersion analysis; mimetic discretization; wave propagation; spurious modes; triangular grid

Software:

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References:

[1] Arakawa, A.; Lamb, V. R., A potential enstrophy and energy conserving scheme for the shallow water equations, Mon. Weather Rev., 109, 18-36, (1981)

[2] Zhang, Y. L.; Baptista, A. M.; Myers, E. P., A cross-scale model for 3D baroclinic circulation in estuary-plume-shelf systems: I. formulation and skill assessment, Cont. Shelf Res., 24, 2187-2214, (2004)

[3] Casulli, V.; Walters, R. A., An unstructured grid, three-dimensional model based on the shallow water equations, Int. J. Numer. Methods Fluids, 32, 331-348, (2000) · Zbl 0965.76061

[4] Danilov, S., On utility of triangular C-grid type discretization for numerical modeling of large-scale Ocean flows, Ocean Dyn., 60, 6, 1361-1369, (2010)

[5] Fringer, O. B.; Gerritsen, M.; Street, R. L., An unstructured-grid, finite-volume, nonhydrostatic, parallel coastal Ocean simulator, Ocean Model., 14, 139-173, (2006)

[6] Gasmann, A., Inspection of hexagonal and triangular C-grid discretizations of the shallow water equations, J. Comput. Phys., 230, 2706-2721, (2011) · Zbl 1316.76069

[7] P. Korn, Formulation of an unstructured grid model for global ocean dynamics, in review. · Zbl 1380.65275

[8] P. Korn, L. Linardakis, A class of potential enstrophy and energy conserving discretizations of the shallow-water equations on unstructured grids, in preparation.

[9] Kleptsova, O.; Pietrzak, J. D.; Stelling, G. S., On the accurate and stable reconstruction of tangential velocities in C-grid

Ocean models, Ocean Model., 28, 118-126, (2009)

[10] Le Roux, D. Y.; Rostand, V.; Pouliot, B., Analysis of numerically induced oscillations in 2D finite-element shallow-water models. part I: inertia-gravity waves, SIAM J. Sci. Comput., 29, 331-360, (2007) · Zbl 1387.76055

[11] Perot, B., Conservation properties of unstructured staggered mesh schemes, J. Comput. Phys., 159, 58-89, (2000) · Zbl 0972.76068

[12] Raviart, P. A.; Thomas, J. M., A mixed finite element method for 2nd order elliptic problems, (Galligani, I.; Magenes, E., Mathematical Aspects of the Finite Element Methods, Lecture Notes in Mathematics, (1977), Springer Berlin), 292-315 · Zbl 0362.65089

[13] Ringler, T. D.; Thuburn, J.; Klemp, J. B.; Skamarock, W. C., A unified approach to energy conservation and potential vorticity dynamics for arbitrarily-structured C-grids, J. Comput. Phys., 229, 3065-3090, (2010) · Zbl 1307.76054

[14] Rostand, V.; Le Roux, D. Y.; Carey, G., Kernel analysis of the discretized finite difference and finite element shallow-water models, SIAM J. Sci. Comput., 31, 531-556, (2008) · Zbl 1191.35025

[15] Stuhne, G. R.; Peltier, W. R., A robust unstructured grid discretization for 3-dimensional hydrostatic flows in spherical geometry: a new numerical structure for Ocean general circulation modeling, J. Comput. Phys., 213, 704-729, (2006) · Zbl 1136.86303

[16] Walters, R. A.; Hanert, E.; Pietrzak, J.; Le Roux, D. Y., Comparison of unstructured, staggered grid methods for the shallow water equations, Ocean Model., 28, 106-117, (2009)

[17] Wan, H.; Giorgetta, M. A.; Zängl, G.; Restelli, M.; Majewski, D.; Bonaventura, L.; Fröhlich, K.; Reinert, D.; Rípodas, P.; Kornblueh, L.; Förstner, J., The ICON-1.2 hydrostatic atmospheric dynamical core on triangular grids — part 1: formulation and performance of the baseline version, Geosci. Model Dev., 6, 735-763, (2013)

[18] Wolfram, P. J.; Fringer, O. B., Mitigating horizontal divergence "checker-board" oscillations on unstructured triangular C-grids for nonlinear hydrostatic and nonhydrostatic flows, Ocean Model., 69, 64-78, (2013)

[19] Zängl, G.; Reinert, D.; Ripodas, P.; Baldauf, M., The ICON (icosahedral non-hydrostatic) modelling framework of DWD and MPI-M: description of the non-hydrostatic dynamical core, Q. J. R. Meteorol. Soc., 141, 563-579, (2015)