

**Pugliese, D.; Valiente Kroon, J. A.**

**On the locally rotationally symmetric Einstein-Maxwell perfect fluid.** (English)

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**Summary:** We examine the stability of Einstein-Maxwell perfect fluid configurations with a privileged radial direction by means of a  $1 + 1 + 2$ -tetrad formalism. We use this formalism to cast in a quasilinear symmetric hyperbolic form the equations describing the evolution of the system. This hyperbolic reduction is used to discuss the stability of linear perturbations in some special cases. By restricting the analysis to isotropic fluid configurations, we assume a constant electrical conductivity coefficient for the fluid. As a result of this analysis we provide a complete classification and characterization of various stable and unstable configurations. We find, in particular, that in many cases the stability conditions are strongly determined by the constitutive equations and the electric conductivity. A threshold for the emergence of the instability appears in both contracting and expanding systems.

**MSC:**

- 83C15 Exact solutions to problems in general relativity and gravitational theory
- 83C22 Einstein-Maxwell equations
- 83C55 Macroscopic interaction of the gravitational field with matter (hydrodynamics, etc.)
- 53Z05 Applications of differential geometry to physics
- 76W05 Magnetohydrodynamics and electrohydrodynamics

**Keywords:**

locally rotationally symmetric solutions;  $1 + 1 + 2$ -formalism; linear perturbations; stability; magnetohydrodynamics

**Full Text:** [DOI](#)

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