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Numerical solutions for solving a class of fractional optimal control problems via fixed-point approach. (English) Zbl 1381.49031

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Summary: In this paper, an optimization problem is performed to obtain an approximate solution for a class of Fractional Optimal Control Problems (FOCPs) with the initial and final conditions. The main characteristic of our approximation is to reduce the FOCP into a system of Volterra integral equations. Then, by solving this new problem, based on minimization and control the total error, we transform the original FOCP into a discrete optimization problem. By obtaining the optimal solutions of this problem, we obtain the numerical solution of the original problem. This procedure not only simplifies the problem but also speeds up the computations. The numerical solutions obtained from the proposed approximation indicate that this approach is easy to implement and accurate when applied to FOCPs.

MSC:

49M25 Discrete approximations in optimal control
49L99 Hamilton-Jacobi theories
65L03 Numerical methods for functional-differential equations
34A08 Fractional ordinary differential equations and fractional differential inclusions
47H10 Fixed-point theorems

Keywords:

Riemann-Liouville fractional derivative; fractional optimal control problem; fractional differential equation; Volterra-integral equation

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