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Nonlinear fractional optimal control problems with neural network and dynamic optimization schemes. (English) Zbl 1377.93078

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Summary: This paper deals with a numerical technique for fractional optimal control problems (FOCPs) based on a neural network scheme. The fractional derivative in these problems is in the Riemann-Liouville sense. The fractional derivative is approximated using the Grunwald-Letnikov definition for numerical computation. According to the Pontryagin's minimum principle (PMP) for FOCPs and by constructing an error function, we define an unconstrained minimization problem. In the optimization problem, we use trial solutions for the state, costate and control functions where these trial solutions are constructed by using two-layered perceptron. We then minimize the error function where weights and biases associated with all neurons are unknown. Substituting the optimal values of the weights and biases in the trial solutions, we obtain the optimal solution of the original problem. Illustrative examples are included to demonstrate the validity and capability of the proposed method.

MSC:

93C10 Nonlinear systems in control theory
92B20 Neural networks for/in biological studies, artificial life and related topics
34A08 Fractional ordinary differential equations and fractional differential inclusions

Keywords:

Pontryagin's minimum principle; fractional optimal control problem; Grunwald-Letnikov operators; artificial neural networks; unconstrained optimization; stability; convergence

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