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Implementation of Newton's method with an analytical Jacobian to solve the 1D sea ice momentum equation. (English) [Zbl 1380.65202](#)

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Summary: New numerical solvers are being considered in response to the rising computational cost of properly solving the sea ice momentum equation at high resolution. The Jacobian free version of Newton's method has allowed models to obtain the converged solution faster than other implicit solvers used previously. To further improve on this recent development, the analytical Jacobian of the 1D sea ice momentum equation is derived and used inside Newton's method. The results are promising in terms of computational efficiency. Although robustness remains an issue for some test cases, it is improved compared to the Jacobian free approach. In order to make use of the strong points of both the new and Jacobian free methods, a hybrid preconditioner using the Picard and Jacobian matrices to improve global and local convergence, respectively, is also introduced. This preconditioner combines the robustness and computational efficiency of the previously used preconditioning matrices when solving the sea ice momentum equation.

MSC:

- 65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
- 65F08 Preconditioners for iterative methods
- 35Q86 PDEs in connection with geophysics

Keywords:

sea ice; Jacobian; viscous-plastic rheology; Newton-Krylov method; numerical convergence

Software:

neXtSIM

Full Text: [DOI](#)

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