

Bergamasco, Adalberto; Parmeggiani, Alberto; Zani, Sérgio; Zugliani, Giuliano
Classes of globally solvable involutive systems. (English) Zbl 1382.58018
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Let M be a smooth closed orientable surface and d the de Rham operator on it. Let \mathbb{S}^1 be the unit circle with the standard unit vector field ∂ . Let b be a closed non-exact 1-form on M . Consider the following complex

$$0 \rightarrow \mathcal{D}'(M \times \mathbb{S}^1) \xrightarrow{\mathbb{L}} \bigwedge^1 \mathcal{D}'(M \times \mathbb{S}^1) \xrightarrow{\mathbb{L}^1} \bigwedge^2 \mathcal{D}'(M \times \mathbb{S}^1) \rightarrow 0,$$

where both operators \mathbb{L} and \mathbb{L}^1 are defined by the formula

$$\mathbb{L}u = du + ib \wedge \partial u.$$

The operator \mathbb{L} defines a first order overdetermined system of linear partial differential equations, whose (local) compatibility conditions are given by the operator \mathbb{L}^1 .

The main theorem states that the global solvability of \mathbb{L} is equivalent to the condition that no super- or sub-level of a potential B of the pull-back of b to the universal cover of M has a bounded component. Moreover, on a certain covering space \tilde{M} , called minimal covering space, the condition is equivalent to the connectedness of the superlevel and sublevel sets of the corresponding pseudo-periodic potential \tilde{B} .

Some versions of this problem have been investigated before in the works of F. Tréves, F. Cardoso, J. Hounie and others.

Reviewer: [Boris S. Kruglikov \(Tromsø\)](#)

MSC:

[58J10](#) Differential complexes

[35A01](#) Existence problems for PDEs: global existence, local existence, non-existence

[35N10](#) Overdetermined systems of PDEs with variable coefficients

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References:

- [1] Arnol'd, VI, Topological and ergodic properties of closed 1-forms with incommensurable periods, *Funkt. Anal. Prilozhen.*, 25, 1-12, (1991) · [Zbl 0732.58001](#)
- [2] Bergamasco, AP; Kirilov, A, Global solvability for a class of overdetermined systems, *J. Funct. Anal.*, 252, 603-629, (2007) · [Zbl 1158.58011](#)
- [3] Bergamasco, AP; Zani, SL, Global analytic regularity for structures of co-rank one, *Commun. Partial Differ. Equ.*, 33, 933-941, (2008) · [Zbl 1153.35006](#)
- [4] Bergamasco, AP; Cordaro, PD; Malagutti, PA, Globally hypoelliptic systems of vector fields, *J. Funct. Anal.*, 114, 267-285, (1993) · [Zbl 0777.58041](#)
- [5] Bergamasco, AP; Cordaro, PD; Petronilho, G, Global solvability for certain classes of underdetermined systems of vector fields, *Math. Z.*, 223, 261-274, (1996) · [Zbl 0863.58062](#)
- [6] Bergamasco, AP; Kirilov, A; Nunes, WVL; Zani, SL, On the global solvability for overdetermined systems, *Trans. Am. Math. Soc.*, 364, 4533-4549, (2012) · [Zbl 1275.35004](#)
- [7] Bergamasco, AP; Medeira, C; Zani, SL, Globally solvable systems of complex vector fields, *J. Differ. Equ.*, 252, 4598-4623, (2012) · [Zbl 1242.35092](#)
- [8] Bergamasco, AP; Nunes, WVL; Zani, SL, Global properties of a class of overdetermined systems, *J. Funct. Anal.*, 200, 31-64, (2003) · [Zbl 1034.32024](#)
- [9] Berhanu, S., Cordaro, P.D., Hounie, J.: *An Introduction to Involutive Structures*. New Mathematical Monographs, vol. 6.

- Cambridge University Press, Cambridge (2008). doi:10.1017/CBO9780511543067 · [Zbl 1151.35011](#)
- [10] Cardoso, F; Hounie, J, Global solvability of an abstract complex, Proc. Am. Math. Soc., 65, 117-124, (1977) · [Zbl 0335.58015](#)
 - [11] Farber, M.: Topology of Closed One-Forms. Mathematical Surveys and Monographs, vol. 108. American Mathematical Society, Providence (2004). doi:10.1090/surv/108 · [Zbl 1052.58016](#)
 - [12] Fenn, R, What is the geometry of a surface?, Am. Math. Mon., 90, 87-98, (1983) · [Zbl 0528.57006](#)
 - [13] Hatcher, A.: Algebraic Topology. Cambridge University Press, Cambridge (2002) · [Zbl 1044.55001](#)
 - [14] Hounie, J., Zugliani, G.: Global solvability of real analytic involutive systems on compact manifolds. Math. Ann. 1-33 (2016). doi:10.1007/s00208-016-1471-5 · [Zbl 1380.35129](#)
 - [15] Jost, J.: Compact Riemann Surfaces, An Introduction to Contemporary Mathematics. Universitext, 3rd edn. Springer, Berlin (2006). doi:10.1007/978-3-540-33067-7 · [Zbl 1125.30033](#)
 - [16] Palais, RS, Natural operations on differential forms, Trans. Am. Math. Soc., 92, 125-141, (1959) · [Zbl 0092.30802](#)
 - [17] Trèves, F.: Topological Vector Spaces, Distributions and Kernels. Academic Press, New York (1967) · [Zbl 0171.10402](#)
 - [18] Treves, F, Study of a model in the theory of complexes of pseudodifferential operators, Ann. Math., 104, 269-324, (1976) · [Zbl 0354.35067](#)
 - [19] Trèves, F.: Hypo-Analytic Structures, Local Theory. Princeton Mathematical Series, vol. 40. Princeton University Press, Princeton (1992) · [Zbl 0787.35003](#)

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