Wu, Jiahong; **Xu, Xiaojing**; **Ye, Zhuan**

Global regularity for several incompressible fluid models with partial dissipation. (English)

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Summary: This paper examines the global regularity problem on several 2D incompressible fluid models with partial dissipation. They are the surface quasi-geostrophic (SQG) equation, the 2D Euler equation and the 2D Boussinesq equations. These are well-known models in fluid mechanics and geophysics. The fundamental issue of whether or not they are globally well-posed has attracted enormous attention. The corresponding models with partial dissipation may arise in physical circumstances when the dissipation varies in different directions. We show that the SQG equation with either horizontal or vertical dissipation always has global solutions. This is in sharp contrast with the inviscid SQG equation for which the global regularity problem remains outstandingly open. Although the 2D Euler is globally well-posed for sufficiently smooth data, the associated equations with partial dissipation no longer conserve the vorticity and the global regularity is not trivial. We are able to prove the global regularity for two partially dissipated Euler equations. Several global bounds are also obtained for a partially dissipated Boussinesq system.

MSC:

35Q35 PDEs in connection with fluid mechanics
35B45 A priori estimates in context of PDEs
35B65 Smoothness and regularity of solutions to PDEs
76D03 Existence, uniqueness, and regularity theory for incompressible viscous fluids
76D09 Viscous-inviscid interaction
35Q31 Euler equations
86A05 Hydrology, hydrography, oceanography
26A33 Fractional derivatives and integrals

Keywords:

Euler equation; surface quasi-geostrophic equation; Boussinesq equations; partial dissipation; global regularity

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