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Generalized matroids based on three-way decision models. (English) Zbl 1419.68170

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Summary: Three-way decision theory is an extension of the commonly used binary-decision model with an added third option. It is originally introduced to explain the three regions of probabilistic rough sets. Every object in a three-way decision model can be assigned to one of the three regions according to its evaluation value under an evaluation function. This paper first introduces three-way decision models based on subset-evaluation which generalize the original models. By the axiomatic approach, we characterize a matroid in terms of evaluation function and then define three-way matroids based on this characterization. Furthermore, three-way matroids are generalized to three-way fuzzy matroids and an equivalent description of three-way fuzzy matroid in terms of fuzzy independent set system is presented. Finally, we give the second description of three-way fuzzy matroid: a three-way fuzzy matroid is exactly the greatest element of an equivalence class. Additionally, relations of notions introduced in this paper are also pointed out.

MSC:

68T37 Reasoning under uncertainty in the context of artificial intelligence
03E72 Theory of fuzzy sets, etc.
05B35 Combinatorial aspects of matroids and geometric lattices

Keywords:

rough sets; three-way decisions; three-way matroids; three-way fuzzy matroids

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