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Global solvability of real analytic involutive systems on compact manifolds. (English)

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Let b be a real analytic closed non-exact 1-form defined on a compact and without boundary connected n -dimensional manifold M , ($n > 1$). The focus of this work is the smooth global solvability of the differential operator $\mathbb{L} : C^\infty(M \times \mathbb{S}^1) \rightarrow \Lambda^1 C^\infty(M \times \mathbb{S}^1)$ given by $\mathbb{L}u = d_{t_j}u + ib(t) \wedge \partial_x u$, where $x \in \mathbb{S}^1$, and d_t is the exterior derivative on M .

The approach relies on defining an appropriate covering projection $\tilde{M} \rightarrow M$ such that the pullback of b has a primitive \tilde{B} and prove that the operator is globally solvable if and only if the superlevel and sublevel sets of \tilde{B} are connected. In case of orientable manifolds M , further characterizations are made.

Reviewer: Marius Ghergu (Dublin)

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35N10 Overdetermined systems of PDEs with variable coefficients

58J10 Differential complexes

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