Harlander, Jens; **Rosebrock, Stephan**

Injective labeled oriented trees are aspherical. (English) Zbl 1381.57002

Math. Z. 287, No. 1-2, 199-214 (2017).

A labeled oriented tree, or LOT, is an oriented graph without cycles, additionally equipped with a labeling map from its set of edges to its set of vertices. In this way, every edge of a LOT has a source, a target, and a label. Associated to every LOT Γ there is a group presentation $P(\Gamma)$, and its corresponding standard 2-complex K_Γ, which is called a LOT complex. LOTs are combinatorial models of ribbon disc complements, generalizing the Wirtinger presentations for classical knots. Knot complements have been proved to be aspherical (see [*C. D. Papakyriakopoulos*, Ann. Math. (2) 66, 1–26 (1957; Zbl 0078.16402)]) and ribbon disc complements are conjectured to be aspherical as well. This problem is an important case of the Whitehead conjecture, which states that subcomplexes of aspherical 2-complexes are aspherical (see [*J. Howie*, Topology 22, 475–485 (1983; Zbl 0524.57002); *S. Rosebrock*, Sib. Èlektron. Mat. Izv. 4, 440–449 (2007; Zbl 1299.57005)]).

The authors introduce a relative version of the notion of vertex asphericity for 2-complexes, and prove that if a 2-complex K is vertex aspherical relative to a subcomplex K_0 and K_0 is aspherical, then K is aspherical. They also provide a test for relative vertex asphericity, based on a result of Stallings. Using these notions, the main result of the article is proved: injective LOTs are aspherical. A LOT is called injective if every vertex occurs at most once as an edge label. Injective LOTs are the analogues, in this context, to alternating knots. They also prove that an inclusion of injective LOTs induces an injective map on their fundamental groups. Finally the authors extend their main result to the non-injective case.

Reviewer: Manuela Cerdeiro (Buenos Aires)

MSC:

57M20 Two-dimensional complexes (manifolds) (MSC2010)
57M35 Dehn's lemma, sphere theorem, loop theorem, asphericity (MSC2010)
57M05 Fundamental group, presentations, free differential calculus
20F05 Generators, relations, and presentations of groups

Keywords:

labeled oriented tree; Wirtinger presentation; 2-complex; asphericity

Full Text: DOI arXiv

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