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Neighbor sum distinguishing total coloring of 2-degenerate graphs. (English) Zbl 1394.05037

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A proper k-total coloring of a graph G is a mapping from $V(G) \cup E(G)$ to $\{1, 2, \ldots, k\}$ such that no two adjacent or incident elements in $V(G) \cup E(G)$ receive the same color. Let $f(v)$ denote the sum of the colors on the edges incident with v and the color on vertex v. A proper k-total coloring of G is called neighbor sum distinguishing if $f(u) \neq f(v)$ for each edge $uv \in E(G)$. The smallest number k in the neighbor sum distinguishing k-total coloring of G is the neighbor sum distinguishing total chromatic number. *M. Pilśniak* and *M. Woźniak* [Graphs Comb. 31, No. 3, 771–782 (2015; Zbl 1312.05054)] conjectured that for any graph G the neighbor sum distinguishing total chromatic number is at most $\Delta(G) + 3$. In this paper, the authors confirm this conjecture for 2-degenerate graphs. Moreover, they improve this bound for graphs with maximum degree at least 5. They prove that if G is 2-degenerate with $\Delta(G) \geq 5$ then the neighbor sum distinguishing total chromatic number is at most $\Delta(G) + 2$. The proof is based on the combinatorial Nullstellensatz. Recently, *L. Ding* et al. [ibid. 33, No. 4, 885–900 (2017; Zbl 1371.05078)] proved that if G is not a forest and $\Delta(G) \geq 4$ then the neighbor sum distinguishing total chromatic number of G is at most $\Delta(G) + 2\mathrm{col}(G) - 1$, where $\mathrm{col}(G)$ is the coloring number of G, in particular, the neighbor sum distinguishing total chromatic number of 2-degenerate graph G with $\Delta(G) \geq 4$ is at most $\Delta(G) + 3$.

Reviewer: Elzbieta Sidorowicz (Zielona Góra)

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