Ding, Laihao; **Wang, Guanghui**; **Wu, Jianliang**; **Yu, Jiguo**

Neighbor sum (set) distinguishing total choosability via the combinatorial Nullstellensatz.
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Summary: Let $G = (V, E)$ be a graph and $\phi : V \cup E \to \{1, 2, \ldots, k\}$ be a total coloring of G. Let $C(v)$ denote the set of the color of vertex v and the colors of the edges incident with v. Let $f(v)$ denote the sum of the color of vertex v and the colors of the edges incident with v. The total coloring ϕ is called neighbor set distinguishing or adjacent vertex distinguishing if $C(u) \neq C(v)$ for each edge $uv \in E(G)$. We say that ϕ is neighbor sum distinguishing if $f(u) \neq f(v)$ for each edge $uv \in E(G)$. In both problems the challenging conjectures presume that such colorings exist for any graph G if $k \geq \Delta(G) + 3$. In this paper, by using the famous Combinatorial Nullstellensatz, we prove that in both problems $k \geq \Delta(G) + 2\mathrm{col}(G) - 2$ is sufficient, moreover we prove that if G is not a forest and $\Delta \geq 4$, then $k \geq \Delta(G) + 2\mathrm{col}(G) - 3$ is sufficient, where $\mathrm{col}(G)$ is the coloring number of G. In fact we prove these results in their list versions, which improve the previous results. As a consequence, we obtain an upper bound of the form $\Delta(G) + C$ for some families of graphs, e.g. $\Delta + 9$ for planar graphs. In particular, we therefore obtain that when $\Delta \geq 4$ two conjectures we mentioned above hold for 2-degenerate graphs (with coloring number at most 3) in their list versions.

MSC:

05C15 Coloring of graphs and hypergraphs

Keywords:

neighbor sum distinguishing total coloring; coloring number; combinatorial Nullstellensatz; list total coloring

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