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Complexity of the improper twin edge coloring of graphs. (English) Zbl 1371.05067

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Summary: Let G be a graph whose each component has order at least 3. Let $s : E(G) \to \mathbb{Z}_k$ for some integer $k \geq 2$ be an improper edge coloring of G (where adjacent edges may be assigned the same color). If the induced vertex coloring $c : V(G) \to \mathbb{Z}_k$ defined by $c(v) = \sum_{e \in E_v} s(e)$ in $\mathbb{Z}_k$, (where the indicated sum is computed in $\mathbb{Z}_k$ and E_v denotes the set of all edges incident to v) results in a proper vertex coloring of G, then we refer to such a coloring as an improper twin k-edge coloring. The minimum k for which G has an improper twin k-edge coloring is called the improper twin chromatic index of G and is denoted by $\chi'_{it}(G)$. It is known that $\chi'_{it}(G) = \chi(G)$, unless $\chi(G) \equiv 2 \pmod 4$ and in this case $\chi'_{it}(G) \in \{\chi(G), \chi(G) + 1\}$. In this paper, we first give a short proof of this result and we show that if G admits an improper twin k-edge coloring for some positive integer k, then G admits an improper twin t-edge coloring for all $t \geq k$; we call this the monotonicity property. In addition, we provide a linear time algorithm to construct an improper twin edge coloring using at most $k + 1$ colors, whenever a k-vertex coloring is given. Then we investigate, to the best of our knowledge the first time in literature, the complexity of deciding whether $\chi'_{it}(G) = \chi(G)$ or $\chi'_{it}(G) = \chi(G) + 1$, and we show that, just like in case of the edge chromatic index, it is NP-hard even in some restricted cases. Lastly, we exhibit several classes of graphs for which the problem is polynomial.

MSC:

05C15 Coloring of graphs and hypergraphs

68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)

Keywords:

modular chromatic index; twin edge/vertex coloring; odd/even color classes; NP-hardness

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