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**Patterns in Calabi-Yau distributions.** (English) Zbl 1376.81059  
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**Summary:** We explore the distribution of topological numbers in Calabi-Yau manifolds, using the Kreuzer-Skarke dataset of hypersurfaces in toric varieties as a testing ground. While the Hodge numbers are well-known to exhibit mirror symmetry, patterns in frequencies of combination thereof exhibit striking new patterns. We find pseudo-Voigt and Planckian distributions with high confidence and exact fit for many substructures. The patterns indicate typicality within the landscape of Calabi-Yau manifolds of various dimension.

**MSC:**

- 81T30** String and superstring theories; other extended objects (e.g., branes) in quantum field theory Cited in 2 Documents
- 14J32** Calabi-Yau manifolds (algebraic-geometric aspects)
- 14J33** Mirror symmetry (algebraic-geometric aspects)

**Software:**

[Calabi-Yau database](#); [cohomCalc](#); [OEIS](#); [PALP](#)

**Full Text:** [DOI](#)

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