Bangert, V.
On minimal laminations of the torus. (English) Zbl 0678.58014
Ann. Inst. Henri Poincaré, Anal. Non Linéaire 6, No. 2, 95-138 (1989).

Consider the problem $\int F(x, u(x), u_x(x))dx \to \min$ where $u : \mathbb{R}^n \to \mathbb{R}$, $F : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is periodic in $(x, u) \in \mathbb{R}^{n+1}$ and uniformly convex in $u_x \in \mathbb{R}^n$. Non-selfintersecting minimizers u are investigated, i.e. the hypersurface graph $(u) \subseteq \mathbb{R}^{n+1}$ has no selfinteractions when projected into T^{n+1}, where T^{n+1} denotes the torus $\mathbb{R}^{n+1}/\mathbb{Z}^{n+1}$. There exists a "rotation vector" $\alpha = \alpha(u) \in \mathbb{R}^n$ for such u so that $u(x) - \alpha$ is bounded uniformly for all $x \in \mathbb{R}^n$. The structure of the set $\mathcal{M}_\alpha = \mathcal{M}_\alpha(F)$ of non-selfintersecting F-minimal solutions with fixed rotation vector α is determined for rationally dependent $\bar{\alpha} = (-\alpha, 1)$. These investigations are primarily topological. $u \in \mathcal{M}_\alpha$ are classified by secondary invariants. The proved uniqueness results mean that the graphs of functions in $\mathcal{M}_\alpha$ with the same secondary invariants do not intersect.

Using these results and the results by *J. Moser* [Ann. Inst. Henri Poincaré, Anal. Non Linéaire 3, 229-272 (1986; Zbl 0609.49029)], the existence of minimal solutions $u \in \mathcal{M}_\alpha$ with prescribed secondary invariants is proved, particularly the existence of secondary laminations in the gaps between the functions in $\mathcal{M}_\alpha$ with maximal periodicity. Further, two open problems are mentioned.

Reviewer: L.Bakule

MSC:

58E15 Variational problems concerning extremal problems in several variables; Yang-Mills functionals
37C85 Dynamics induced by group actions other than $\mathbb{Z}$ and $\mathbb{R}$, and $\mathbb{C}$
49Q20 Variational problems in a geometric measure-theoretic setting

Keywords:

Z^n-periodic variational problem; minimizing solutions; laminations

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