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Proof of a conjecture about unimodal polynomials. (English) Zbl 0678.05002

The author investigates the well-known conjecture that the polynomial \( \prod_{r=1}^{n} (1-t^{n-r^2})/(1-t^r) \) is symmetric unimodal if \( n \) is even and \( r \geq 1 \), or \( n \) is odd and \( r \geq 11 \). It is proved that conjecture is true for \( 3 \leq n \leq 20 \) and for \( n = 100 \) and \( n = 101 \). The method of proof makes use of a clever induction for “large” \( r \), say \( r \geq r_0(n) \), where \( r_0(n) \approx 10 \) for large \( n \). As by-product in the course of proof, two elegant inequalities for Chebyshev polynomials are obtained, namely \( |U_{n-1}(x)| \leq n^2x^2 \) and \( |T_n'(x)| \leq n^{1+x^2} (|x| \leq 1) \).

Reviewer: L.C.Hsu

MSC:
05A10 Factorials, binomial coefficients, combinatorial functions
05A17 Combinatorial aspects of partitions of integers
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Keywords:
unimodal polynomial; number of restrictive partitions; Chebyshev polynomials

Full Text: DOI

References:
[1] Abramowitz, M.; Segun, I.A., ()

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