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Gauge-theoretic invariants for topological insulators: a bridge between Berry, Wess-Zumino, and Fu-Kane-Mele. (English) Zbl 1370.35093

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Summary: We establish a connection between two recently proposed approaches to the understanding of the geometric origin of the Fu-Kane-Mele invariant FKM $\in \mathbb{Z}_2$, arising in the context of two-dimensional time-reversal symmetric topological insulators. On the one hand, the $\mathbb{Z}_2$ invariant can be formulated in terms of the Berry connection and the Berry curvature of the Bloch bundle of occupied states over the Brillouin torus. On the other, using techniques from the theory of bundle gerbes, it is possible to provide an expression for FKM containing the square root of the Wess-Zumino amplitude for a certain $U(N)$-valued field over the Brillouin torus. We link the two formulas by showing directly the equality between the above-mentioned Wess-Zumino amplitude and the Berry phase, as well as between their square roots. An essential tool of independent interest is an equivariant version of the adjoint Polyakov-Wiegmann formula for fields $\mathbb{T}^2 \to U(N)$, of which we provide a proof employing only basic homotopy theory and circumventing the language of bundle gerbes.

MSC:

35J10 Schrödinger operator, Schrödinger equation
81Q30 Feynman integrals and graphs; applications of algebraic topology and algebraic geometry
81Q70 Differential geometric methods, including holonomy, Berry and Hannay phases, Aharonov-Bohm effect, etc. in quantum theory

Keywords:

time-reversal symmetric topological insulators; Fu-Kane-Mele $\mathbb{Z}_2$ invariant; Wess-Zumino amplitude; Berry connection; Polyakov-Wiegmann formula

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