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A new formula for the decycling number of regular graphs. (English) Zbl 1370.05105
Discrete Math. 340, No. 12, 3020-3031 (2017).

Summary: The decycling number $\nabla(G)$ of a graph G is the smallest number of vertices which can be removed from G so that the resultant graph contains no cycle. A decycling set containing exactly $\nabla(G)$ vertices of G is called a ∇ -set. For any decycling set S of a k -regular graph G , we show that $|S| = \frac{\beta(G)+m(S)}{k-1}$, where $\beta(G)$ is the cycle rank of G , $m(S) = c + |E(S)| - 1$ is the margin number of S , c and $|E(S)|$ are, respectively, the number of components of $G - S$ and the number of edges in $G[S]$. In particular, for any ∇ -set S of a 3-regular graph G , we prove that $m(S) = \xi(G)$, where $\xi(G)$ is the Betti deficiency of G . This implies that the decycling number of a 3-regular graph G is $\frac{\beta(G)+\xi(G)}{2}$. Hence $\nabla(G) = \lceil \frac{\beta(G)}{2} \rceil$ for a 3-regular upper-embeddable graph G , which concludes the results in [L. Gao et al., Discrete Appl. Math. 181, 297–300 (2015; Zbl 1304.05082); E. Wei and Y. Li, Acta Math. Sin., Chin. Ser. 56, No. 2, 211–216 (2013; Zbl 1289.05107)] and solves two open problems posed by S. Bau and L. W. Beineke [Australas. J. Comb. 25, 285–298 (2002; Zbl 0994.05079)]. Considering an algorithm by M. Furst, J. L. Gross and L. A. McGeoch [“Finding a maximum genus graph imbedding”, J. Assoc. Comput. Mach. 35, No. 3, 523–534 (1988; doi:10.1145/44483.44485)], there exists a polynomial time algorithm to compute $Z(G)$, the cardinality of a maximum nonseparating independent set in a 3-regular graph G , which solves an open problem raised by E. Speckenmeyer [J. Graph Theory 12, No. 3, 405–412 (1988; Zbl 0657.05042)]. As for a 4-regular graph G , we show that for any ∇ -set S of G , there exists a spanning tree T of G such that the elements of S are simply the leaves of T with at most two exceptions providing $\nabla(G) = \lceil \frac{\beta(G)}{3} \rceil$. On the other hand, if G is a loopless graph on n vertices with maximum degree at most 4, then

$$\nabla(G) \leq \begin{cases} \frac{n+1}{2}, & \text{if } G \text{ is 4-regular,} \\ \frac{n}{2}, & \text{otherwise.} \end{cases}$$

The above two upper bounds are tight, and this makes an extension of a result due to N. Punnim [Thai J. Math. 4, No. 1, 145–161 (2006; Zbl 1156.05317)].

MSC:

05C35 Extremal problems in graph theory

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References:

- [1] M. Albertson, D. Berman, The acyclic chromatic number, Congr. Numer., 17(1976), 51-69.. · [Zbl 0344.05116](#)
- [2] S. Bau, L. Beineke, The decycling number of graphs, Australas. J. Combin., 25(2002), 285-298.. · [Zbl 0994.05079](#)
- [3] S. Bau, N. Wormald, S. Zhou, Decycling numbers of random regular graphs, Random Structures Algorithms, 21(2002), no. 3-4, 397-413.. · [Zbl 1012.05099](#)
- [4] L. Beineke, R. Vandell, Decycling graphs, J. Graph Theory, 25(1997), no. 1, 59-77.. · [Zbl 0870.05033](#)
- [5] R. Diestel, Graph Theory, Springer-Verlag, New York, 1997..
- [6] P. Erdős, M. Saks, V. Sós, Maximum induced trees in graphs, J. Combin. Theory Ser. B, 41(1986), 61-79..
- [7] R.Focardi, F.Luccio, D.Peleg, Feedback vertex set in hypercubes, Inform.Process.Lett., 76(2000), no.1-2, 1-5.. · [Zbl 1338.68218](#)
- [8] M.Furst, J.Gross, L.Mcgeoch, Finding a maximum genus graph imbedding, J.Assoc.Comput.Mach, 35(1988), no.3, 523-534..
- [9] L.Gao, X.Xu, J.Wang, D.Zhu, Y.Yang, The decycling number of generalized Petersen graphs, Discrete Appl. Math., 181(2015), 297C300.. · [Zbl 1304.05082](#)

- [10] F. Harary, Graph Theory, Academic Press, New York, 1967..
- [11] G.Kirchhoff, Über die Auflösung der Gleichungen, auf welche man bei der Untersuchung der linearen Verteilung galvanischer Ströme geführt wird, Ann. Phys. Chem., 72(1847), 497-508..
- [12] R.Karp, Reducibility among combinatorial problems, Complexity of computer computations, (Proc. Sympos., IBM Thomas J.Watson Res.Center, Yorktown Heights, N.Y., 1972), 85C103.Plenum, New York, 1972.. · [Zbl 1187.90014](#)
- [13] Y.Liu, The maximum orientable genus of a graph, Sci.Sinica, Special Issue on Math., 1979, Special Issue II on Math., 41-55..
- [14] D.Ma, Embedding of graphs on surface and crossing numbers, Master thesis, Shanghai, East China Normal University, 2004..
- [15] D.Pike, Y.Zou, Decycling cartesian products of two cycles, SIAM J.Discrete Math., 19(2005), no.3, 651-663.. · [Zbl 1096.05030](#)
- [16] N. Punnim, The decycling number of cubic graphs, Combinatorial Geometry and Graph Theory, 141-145, Lecture Notes in Comput. Sci., 3330, Springer, Berlin, 2005.. · [Zbl 1117.05022](#)
- [17] N.Punnim, The decycling number of cubic planar graphs, Discrete Geometry, Combinatorics and Graph Theory, 149-161, Lecture Notes in Comput. Sci., 4381, Springer, Berlin, 2007.. · [Zbl 1149.05315](#)
- [18] N. Punnim, The decycling number of regular graphs, Thai J. Math., 4(2006), 145-161.. · [Zbl 1156.05317](#)
- [19] H. Ren, S. Long, The decycling number and maximum genus of cubic graphs. (Submitted). · [Zbl 1393.05166](#)
- [20] E.Speckenmeyer, On feedback vertex sets and nonseparating independent sets in cubic graphs, J. Graph Theory, 12(1988), no. 3, 405-412.. · [Zbl 0657.05042](#)
- [21] E. Wei, Y. Liu, Z. Li, Decycling number of circular graphs, ISORA'09, (2009), 387-393..
- [22] E.Wei, Y.Li, Decycling number and upper-embeddibility of generalized Petersen graphs, Acta Math. Sinica(Chin. Ser.), 56(2013), no. 2, 211-216.. · [Zbl 1289.05107](#)
- [23] N.Xuong, How to determine the maximum genus of a graph, J.Combin.Theory Ser.B, 26(1979), 217-225.. · [Zbl 0403.05035](#)

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