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A decoupled unconditionally stable numerical scheme for the Cahn-Hilliard-Hele-Shaw system. (English) Zbl 06760785

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Summary: We propose a novel decoupled unconditionally stable numerical scheme for the simulation of two-phase flow in a Hele-Shaw cell which is governed by the Cahn-Hilliard-Hele-Shaw system (CHHS) with variable viscosity. The temporal discretization of the Cahn-Hilliard equation is based on a convex-splitting of the associated energy functional. Moreover, the capillary forcing term in the Darcy equation is separated from the pressure gradient at the time discrete level by using an operator-splitting strategy. Thus the computation of the nonlinear Cahn-Hilliard equation is completely decoupled from the update of pressure. Finally, a pressure-stabilization technique is used in the update of pressure so that at each time step one only needs to solve a Poisson equation with constant coefficient. We show that the scheme is unconditionally stable. Numerical results are presented to demonstrate the accuracy and efficiency of our scheme.

MSC:

65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
76T99 Multiphase and multicomponent flows
76D27 Other free boundary flows; Hele-Shaw flows
76S05 Flows in porous media; filtration; seepage
35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation
35Q35 PDEs in connection with fluid mechanics

Keywords:

Cahn-Hilliard-Hele-Shaw; decoupling; unconditional stability; convex-splitting; operator-splitting

Software:

FreeFem++

Full Text: DOI arXiv

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