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Frozen Jacobian multistep iterative method for solving nonlinear IVPs and BVPs. (English)

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Summary: In this paper, we present and illustrate a frozen Jacobian multistep iterative method to solve systems of nonlinear equations associated with Initial Value Problems (IVPs) and Boundary Value Problems (BVPs). We have used Jacobi-Gauss-Lobatto collocation (J-GL-C) methods to discretize the IVPs and BVPs. Frozen Jacobian multistep iterative methods are computationally very efficient. They require only one inversion of the Jacobian in the form of LU-factorization. The LU factors can then be used repeatedly in the multistep part to solve other linear systems. The convergence order of the proposed iterative method is $5m - 11$, where m is the number of steps. The validity, accuracy, and efficiency of our proposed frozen Jacobian multistep iterative method is illustrated by solving fifteen IVPs and BVPs. It has been observed that, in all the test problems, with one exception in this paper, a single application of the proposed method is enough to obtain highly accurate numerical solutions. In addition, we present a comprehensive comparison of J-GL-C methods on a collection of test problems.

MSC:

65H10 Numerical computation of solutions to systems of equations
65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

Keywords:

frozen Jacobian multistep iterative method; systems of nonlinear equations; initial value problems (IVPs); boundary value problems (BVPs); Jacobi-Gauss-Lobatto collocation (J-GL-C) methods; discretization

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