

Mj, Mahan

Cannon-Thurston maps for Kleinian groups. (English) Zbl 1370.57008
Forum Math. Pi 5, Paper No. e1, 49 p. (2017).

Question 14 of Thurston's famous and most influential problem list on hyperbolic 3-manifolds and Kleinian groups can be rephrased as follows. Suppose that Γ is a geometrically finite Kleinian group and G an arbitrary Kleinian group abstractly isomorphic to Γ , by an isomorphism preserving parabolics. Then does there exist a continuous map from the set of limit points of Γ to the set of limit points of G taking a fixed point of an element of Γ to a fixed point of the corresponding element of G ? Such a continuous map is called a *Cannon-Thurston map*, cf. the paper of *J. W. Cannon* and *W. P. Thurston* [Geom. Topol. 11, 1315–1355 (2007; Zbl 1136.57009)], based on a preprint from 1985.

In a previous paper [Ann. Math. (2) 179, No. 1, 1–80 (2014; Zbl 1301.57013)], the present author showed that Cannon-Thurston maps exist for simply or doubly degenerate surface Kleinian groups without accidental parabolics, and in a second paper [Geom. Funct. Anal. 24, No. 1, 297–321 (2014; Zbl 1297.57040)] the author showed that point pre-images of the Cannon-Thurston map for such groups correspond to endpoints of leaves of ending laminations whenever a point has more than one pre-image.

“The aim of this paper is to apply the techniques developed in these two papers to extend these results to arbitrary finitely generated Kleinian groups without parabolics”. “We show that Cannon-Thurston maps exist for degenerate free groups without parabolics, that is, for handlebody groups. Combining these techniques with earlier work proving the existence of Cannon-Thurston maps for surface groups, we show that Cannon-Thurston maps exist for arbitrary finitely generated Kleinian groups without parabolics, proving conjectures of Thurston and McMullen. We also show that point pre-images under Cannon-Thurston maps for degenerate free groups without parabolics correspond to endpoints of leaves of an ending lamination in the Masur domain, whenever a point has more than one pre-image. This proves a conjecture of Otal. We also prove a similar result for point pre-images under Cannon-Thurston maps for arbitrary finitely generated Kleinian groups without parabolics.”

Reviewer: Bruno Zimmermann (Trieste)

MSC:

57M50 General geometric structures on low-dimensional manifolds
30F40 Kleinian groups (aspects of compact Riemann surfaces and uniformization)

Cited in 10 Documents

Keywords:

Kleinian group; Cannon-Thurston map; degenerate group

Full Text: DOI arXiv

References:

- [1] Agol, I., ‘Tameness of hyperbolic 3-manifolds’. Preprint, 2004, arXiv:math.GT/0405568.
- [2] Brock, J. and Bromberg, K., ‘Density of geometrically finite Kleinian groups’, Acta Math.192 (2004), 33-93. doi:10.1007/BF02441085 · Zbl 1055.57020
- [3] Brock, J. and Bromberg, K., ‘Geometric inflexibility and 3-manifolds that fiber over the circle’, J. Topol.4(1) (2011), 1-38. doi:10.1112/jtopol/jtq032 · Zbl 1220.30057
- [4] Brock, J. and Bromberg, K., ‘Geometric inflexibility of hyperbolic cone-manifolds’. Preprint, 2014, \textit{Proceedings of the 2014 MSJ-SI}, arXiv:1412.4635, to appear. · Zbl 1220.30057
- [5] Brock, J., Bromberg, K., Evans, R. and Souto, J., ‘Tameness on the boundary and Ahlfors’ measure conjecture’, Publ. Math. Inst. Hautes Études Sci.98 (2003), 145-166. doi:10.1007/s10240-003-0018-y · Zbl 1060.30054
- [6] Brock, J. F., Canary, R. D. and Minsky, Y. N., ‘The classification of Kleinian surface groups II: the ending lamination conjecture’, Ann. of Math. (2)176(1) (2012), 1-149. doi:10.4007/annals.2012.176.1.1 · Zbl 1253.57009
- [7] Brock, J. F., Canary, R. D. and Minsky, Y. N., ‘The classification of finitely generated Kleinian groups’, in preparation, 2014.

- [8] Bonahon, F., Bouts de varietes hyperboliques de dimension 3, *Ann. of Math. (2)*, 124, 71-158, (1986) · [Zbl 0671.57008](#)
- [9] Bowditch, B. H., ‘Model geometries for hyperbolic manifolds’. Preprint, Southampton, 2005.
- [10] Bowditch, B. H., ‘The ending lamination theorem’. Preprint, Warwick, 2011.
- [11] Bowditch, B. H., ‘Relatively hyperbolic groups’, *Internat. J. Algebra Comput.*22(3) (2012), 1250016, 66 pp. doi:10.1142/S0218196712500166 · [Zbl 1259.20052](#)
- [12] Bowditch, B. H., *Geometry and Topology Down Under*, 65-138, (2013), American Mathematical Society: American Mathematical Society, Providence, RI · [Zbl 1297.57044](#)
- [13] Bromberg, K., Projective structures with degenerate holonomy and the Bers density conjecture, *Ann. of Math. (2)*, 166, 1, 77-93, (2007) · [Zbl 1137.30014](#)
- [14] Canary, R. D., Ends of hyperbolic 3 manifolds, *J. Amer. Math. Soc.*, 6, 1-35, (1993) · [Zbl 0810.57006](#)
- [15] Cannon, J. and Thurston, W. P., *Group invariant Peano Curves*. Preprint, Princeton, 1985. · [Zbl 1136.57009](#)
- [16] Cannon, J. and Thurston, W. P., ‘Group invariant Peano curves’, *Geom. Topol.*11 (2007), 1315-1356. doi:10.2140/gt.2007.11.1315 · [Zbl 1136.57009](#)
- [17] Das, S. and Mj, M., ‘Semiconjugacies between relatively hyperbolic boundaries’, *Groups Geom. Dyn.*10(2) (2016), 733-752. doi:10.4171/GGD/363 · [Zbl 06605290](#)
- [18] Farb, B., Relatively hyperbolic groups, *Geom. Funct. Anal.*, 8, 810-840, (1998) · [Zbl 0985.20027](#)
- [19] Floyd, W. J., Group completions and limit sets of Kleinian groups, *Invent. Math.*, 57, 205-218, (1980) · [Zbl 0428.20022](#)
- [20] Calegari, D. and Gabai, D., ‘Shrink-wrapping and the taming of hyperbolic 3-manifolds’, *J. Amer. Math. Soc.*19(2) (2006), 385-446. doi:10.1090/S0894-0347-05-00513-8 · [Zbl 1090.57010](#)
- [21] Hempel, J., *3 Manifolds*, (1976), Princeton University Press
- [22] Hubbard, J. H., ‘Local connectivity of Julia sets and bifurcation loci: three theorems of J.-C. Yoccoz’, in *Topological Methods in Modern Mathematics* (eds. Goldberg, L. R. and Philips, A. G.) (Publish or Perish Inc., Houston, TX, 1993), 467-511. · [Zbl 0797.58049](#)
- [23] Jeon, W., Kim, I., Lecuire, C. and Ohshika, K., ‘Primitive stable representations of free Kleinian groups’, *Israel J. Math.*199(2) (2014), 841-866. doi:10.1007/s11856-013-0062-3 · [Zbl 1361.57025](#)
- [24] Klarreich, E., Semiconjugacies between Kleinian group actions on the Riemann sphere, *Amer. J. Math.*, 121, 1031-1078, (1999) · [Zbl 1011.30035](#)
- [25] Kleinedam, G. and Souto, J., ‘Ending laminations in the masur domain’, in *Kleinian Groups and Hyperbolic 3-Manifolds* (Warwick, 2001), (eds. Komori, Y., Markovic, V. and Series, C.) (Cambridge University Press, Cambridge, 2003), 105-129. · [Zbl 1052.57020](#)
- [26] Leininger, C. J., Long, D. D. and Reid, A. W., ‘Commensurators of non-free finitely generated Kleinian groups’, *Algebr. Geom. Topol.*11 (2011), 605-624. doi:10.2140/agt.2011.11.605 · [Zbl 1237.20044](#)
- [27] Luft, E., Actions of the homeotopy group of an orientable 3-dimensional handlebody, *Math. Ann.*, 234, 3, 279-292, (1978) · [Zbl 0364.57011](#)
- [28] McMullen, C. T., *Renormalization and 3-manifolds which Fiber over the Circle*, (1998), Princeton University Press: Princeton University Press, Princeton, NJ
- [29] McMullen, C. T., Local connectivity, Kleinian groups and geodesics on the blow-up of the torus, *Invent. Math.*, 97, 95-127, (2001) · [Zbl 0672.30017](#)
- [30] Milnor, J., ‘Local connectivity of Julia sets: expository lectures’, in *The Mandelbrot Set, Theme and Variations* (ed. Lei, T.) (Cambridge University Press, Cambridge, 2000), 67-116. doi:10.1017/CBO9780511569159.006 · [Zbl 1107.37305](#)
- [31] Minsky, Y. N., The classification of Kleinian surface groups I: models and bounds, *Ann. of Math. (2)*, 171, 1-107, (2010) · [Zbl 1193.30063](#)
- [32] Minsky, Y. N., On dynamics of $\text{Out}(\mathbb{F}_n)$ on $\text{PSL}_2(\mathbb{C})$ characters, *Israel J. Math.*, 193, 1, 47-70, (2013) · [Zbl 1282.57023](#)
- [33] Mitra, M., Cannon-Thurston maps for trees of hyperbolic metric spaces, *J. Differential Geom.*, 48, 135-164, (1998) · [Zbl 0906.20023](#)
- [34] Miyachi, H., ‘Semiconjugacies between actions of topologically tame Kleinian groups’. Preprint, 2002.
- [35] Mj, M., On discreteness of commensurators, *Geom. and Topol.*, 15, 331-350, (2011) · [Zbl 1209.57010](#)
- [36] Mj, M., Cannon-Thurston maps for surface groups, *Ann. of Math. (2)*, 179, 1, 1-80, (2014) · [Zbl 1301.57013](#)
- [37] Mj, M., Ending laminations and Cannon-Thurston Maps, with an appendix by S. Das and M. Mj, *Geom. Funct. Anal.*, 24, 297-321, (2014) · [Zbl 1297.57040](#)
- [38] McCullough, D. and Miller, A., ‘Homeomorphisms of 3-manifolds with compressible boundary’, *Mem. Amer. Math. Soc.*61(344) (1986), xii+100 pp. · [Zbl 0602.57011](#)
- [39] Mosher, L., Stable Teichmüller quasigeodesics and ending laminations, *Geom. Topol.*, 7, 33-90, (2003) · [Zbl 1021.57009](#)
- [40] McCarthy, J. D. and Papadopoulos, A., ‘Dynamics on Thurston’s sphere of projective measured foliations’, *Comment. Math. Helv.*64 (1989), 133-166. doi:10.1007/BF02564666 · [Zbl 0681.57002](#)
- [41] Mj, M. and Pal, A., ‘Relative hyperbolicity, trees of spaces and Cannon-Thurston maps’, *Geom. Dedicata*151 (2011), 59-78. doi:10.1007/s10711-010-9519-2 · [Zbl 1222.57013](#)

- [42] Ohshika, K., Rigidity and topological conjugates of topologically tame Kleinian groups, *Trans. Amer. Math. Soc.*, 350, 10, 3989-4022, (1998) · [Zbl 0936.30031](#)
- [43] Otal, J. P., 'Courants geodesiques et produit libres', These d'Etat, Universit Paris-Sud, Orsay, 1988.
- [44] Otal, J. P., Sur le nouage des geodesiques dans les varietes hyperboliques, *C. R. Acad. Sci. Paris Ser. I Math.*, 320, 7, 847-852, (1995) · [Zbl 0840.57008](#)
- [45] Otal, J. P., 'Les geodesiques fermees d'une variete hyperbolique en tant que noeuds', in *Kleinian Groups and Hyperbolic 3-Manifolds* (London Math. Soc., Orsay, 2003). · [Zbl 1049.57007](#)
- [46] Souto, J., 'Cannon-Thurston maps for thick free groups'. Preprint, 2006.
- [47] Souto, J., 'Short geodesics in hyperbolic compression bodies are not knotted'. Preprint, 2008.
- [48] Sullivan, D., 'Conformal dynamical systems', in *Geometric Dynamics*, (Springer, Berlin, 1983), 725-752. doi:10.1007/BFb0061443
- [49] Sullivan, D., Quasiconformal homeomorphisms and dynamics I: solution of the Fatou-Julia problem on wandering domains, *Ann. of Math. (2)*, 122, 401-418, (1985) · [Zbl 0589.30022](#)
- [50] Suzuki, S., On homeomorphisms of a 3-dimensional handlebody, *Canad. J. Math.*, 29, 1, 111-124, (1977) · [Zbl 0339.57001](#)
- [51] Thurston, W. P., *The Geometry and Topology of 3-Manifolds*, (1980), Princeton University Notes: Princeton University Notes, Princeton, NJ
- [52] Thurston, W. P., Three dimensional manifolds, Kleinian groups and hyperbolic geometry, *Bull. Amer. Math. Soc.*, 6, 357-382, (1982) · [Zbl 0496.57005](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.