Chan, Timothy M.; **He, Meng**; **Munro, J. Ian**; **Zhou, Gelin**
Succinct indices for path minimum, with applications. (English) Zbl 1369.68167
Algorithmica 78, No. 2, 453-491 (2017).

Summary: In the *path minimum* problem, we preprocess a tree on n weighted nodes, such that given an arbitrary path, the node with the smallest weight along this path can be located. We design novel succinct indices for this problem under the *indexing model*, for which weights of nodes are read-only and can be accessed with ranks of nodes in the preorder traversal sequence of the input tree. We present

- an index within $O(m)$ bits of additional space that supports queries in $O(\boldsymbol{\alpha}(m,n))$ time and $O(\boldsymbol{\alpha}(m,n))$ accesses to the weights of nodes, for any integer $m \geq n$; and
- an index within $2n + o(n)$ bits of additional space that supports queries in $O(\boldsymbol{\alpha}(n))$ time and $O(\boldsymbol{\alpha}(n))$ accesses to the weights of nodes.

Here $\boldsymbol{\alpha}(m,n)$ is the inverse-Ackermann function, and $\boldsymbol{\alpha}(n) = \boldsymbol{\alpha}(n,n)$. These indices give us the first succinct data structures for the path minimum problem. Following the same approach, we also develop succinct data structures for *semigroup path sum* queries, for which a query asks for the sum of weights along a given query path. One of our data structures requires $n \lg \sigma + 2n + o(n \lg \sigma)$ bits of space and $O(\boldsymbol{\alpha}(n))$ query time, where σ is the size of the semigroup. In the *path reporting* problem, queries ask for the nodes along a query path whose weights are within a two-sided query range. Using the succinct indices for path minimum queries, we achieve three different time/space tradeoffs for path reporting by designing

- an $O(n)$-word data structure with $O(\lg^{\epsilon} n + \mathrm{occ} \cdot \lg^{\epsilon} n)$ query time;
- an $O(n \lg \lg n)$-word data structure with $O(\lg \lg n + \mathrm{occ} \cdot \lg \lg n)$ query time; and
- an $O(n \lg^{\epsilon} n)$-word data structure with $O(\lg \lg n + \mathrm{occ})$ query time.

Here occ is the number of nodes reported and ϵ is an arbitrary constant between 0 and 1. These tradeoffs match the state of the art of two-dimensional orthogonal range reporting queries [the first author et al., in: Proceedings of the 27th annual symposium on computational geometry, SoCG'11. New York, NY: Association for Computing Machinery (ACM). 1–10 (2011; Zbl 1283.68139)], which can be treated as a special case of path reporting queries. When the number of distinct weights is much smaller than n, we further improve both the query time and the space cost of these three results.

MSC:

68P05 Data structures

Cited in **1** Document

Keywords:

path minimum; semigroup path sum; path reporting; succinct data structures; succinct encoding of directed topology trees

Full Text: DOI

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