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Hölder continuous densities of solutions of SDEs with measurable and path dependent drift coefficients. (English) Zbl 1367.60064

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Summary: We consider a process given as the solution of a one-dimensional stochastic differential equation with irregular, path dependent and time-inhomogeneous drift coefficient and additive noise. Hölder continuity of the density at any given time is achieved using a different approach than the classical ones in the literature. Namely, the Hölder regularity is obtained via a control problem by identifying the equation with the worst global Hölder constant. Then we generalise our findings to a larger class of diffusions. The novelty of this method is that it is not based on a variational calculus and it is suitable for non-Markovian processes.

MSC:

60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)

Cited in 4 Documents

49N60 Regularity of solutions in optimal control

Keywords:

stochastic differential equations; densities; regularity; irregular drift; stochastic control

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