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$(1, k)$-coloring of graphs with girth at least five on a surface. (English) Zbl 1359.05099

J. Graph Theory 84, No. 4, 521-535 (2017).

Summary: A graph is $(d_1 \dots, d_r)$-colorable if its vertex set can be partitioned into r sets $V_1, \dots, V_r$ so that the maximum degree of the graph induced by V_i is at most d_i for each $i \in \{1, \dots, r\}$. For a given pair (g, d_1), the question of determining the minimum $d_2 = d_2(g, d_1)$ such that planar graphs with girth at least g are (d_1, d_2)-colorable has attracted much interest. The finiteness of $d_2(g, d_1)$ was known for all cases except when $(g, d_1) = (5, 1)$. *M. Montassier* and *P. Ochem* [Electron. J. Comb. 22, No. 1, Research Paper P1.57, 13 p. (2015; Zbl 1308.05052)] explicitly asked if $d_2(5, 1)$ is finite. We answer this question in the affirmative with $d_2(5, 1) \leq 10$; namely, we prove that all planar graphs with girth at least five are $(1, 10)$-colorable. Moreover, our proof extends to the statement that for any surface S of Euler genus γ, there exists a $K = K(\gamma)$ where graphs with girth at least five that are embeddable on S are $(1, K)$-colorable. On the other hand, there is no finite k where planar graphs (and thus embeddable on any surface) with girth at least five are $(0, k)$-colorable.

MSC:

05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)
05C07 Vertex degrees
05C15 Coloring of graphs and hypergraphs

Keywords:

improper coloring; discharging; graphs on surfaces

Full Text: DOI

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