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On the minimum number of spanning trees in k-edge-connected graphs. (English)

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Summary: We show that a k-edge-connected graph on n vertices has at least $n(k/2)^{n-1}$ spanning trees. This bound is tight if k is even and the extremal graph is the n-cycle with edge multiplicities $k/2$. For k odd, however, there is a lower bound c_k^{n-1}, where $c_k > k/2$. Specifically, $c_3 > 1.77$ and $c_5 > 2.75$. Not surprisingly, c_3 is smaller than the corresponding number for 4-edge-connected graphs. Examples show that $c_3 \leq \sqrt{2+\sqrt{3}} \approx 1.93$. However, we have no examples of 5-edge-connected graphs with fewer spanning trees than the n-cycle with all edge multiplicities (except one) equal to 3, which is almost 6-regular. We have no examples of 5-regular 5-edge-connected graphs with fewer than 3.09^{n-1} spanning trees, which is more than the corresponding number for 6-regular 6-edge-connected graphs. The analogous surprising phenomenon occurs for each higher odd edge connectivity and regularity.

MSC:

05C05 Trees
05C40 Connectivity

Cited in **2** Documents

Keywords:

spanning tree; cubic graph; edge connectivity

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