**Jerrard, Robert L.**; **Seis, Christian**

**On the vortex filament conjecture for Euler flows.** (English) Zbl 1371.35205

Arch. Ration. Mech. Anal. 224, No. 1, 135-172 (2017).

In this paper the authors consider the evolution of a vortex filament in an incompressible ideal fluid. The fluid motion is described by the Euler equation. The authors are mainly interested in the case where the initial vorticity is concentrated in a tube of radius $\varepsilon \ll 1$ around a smooth curve in $\mathbb{R}^3$. Then they pose the following two questions: 1. Does the vorticity continue to concentrate around some curve at later times ?

2. If so, how does the curve evolve ?

It can be proved that the curve evolves to leading order by binormal curvature flow. The used approach combines new estimates on the distance of the corresponding Hamiltonian-Poisson structures with stability estimates recently developed by the first author (partially). The bibliography contains 33 items. The authors give an appropriate overview on the problem. The paper is self-contained and reads good.

Reviewer: Jürgen Socolowsky (Brandenburg an der Havel)

**MSC:**

35Q31 Euler equations
35B45 A priori estimates in context of PDEs
35B35 Stability in context of PDEs
76B03 Existence, uniqueness, and regularity theory for incompressible inviscid fluids

Cited in **8** Documents

**Keywords:**

vortex filament; Euler equations; stability

**Full Text:** DOI arXiv

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