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On a Li-type criterion for zero-free regions of certain Dirichlet series with real coefficients. (English) Zbl 1391.11101

LMS J. Comput. Math. 19, No. 1, 259-280 (2016).

Summary: The Li coefficients $\lambda_F(n)$ of a zeta or L-function F provide an equivalent criterion for the (generalized) Riemann hypothesis. In this paper we define these coefficients, and their generalizations, the τ-Li coefficients, for a subclass of the extended Selberg class which is known to contain functions violating the Riemann hypothesis such as the Davenport-Heilbronn zeta function. The behavior of the τ-Li coefficients varies depending on whether the function in question has any zeros in the half-plane $\mathrm{Re}(z) > \tau/2$. We investigate analytically and numerically the behavior of these coefficients for such functions in both the n and τ aspects.

MSC:

11M26 Nonreal zeros of $\zeta(s)$ and $L(s,\chi)$; Riemann and other hypotheses
11M36 Selberg zeta functions and regularized determinants; applications to spectral theory, Dirichlet series, Eisenstein series, etc. (explicit formulas)
11M41 Other Dirichlet series and zeta functions

Software:

Arb

Full Text: DOI

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