Gorchinskiy, Sergey O.; **Osipov, Denis V.**

Continuous homomorphisms between algebras of iterated Laurent series over a ring. (English. Russian original) Zbl 1359.13023

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Let A be a commutative ring, $A((t))$ be the ring of Laurent series over A and $\mathcal{L}^n(A) = A((t_1))\dots((t_n))$ the $A-$algebra of iterated Laurent series over A with the natural topology. The elements of $\mathcal{L}^n(A)$ have the form $\sum\limits_{l\in\mathbb{Z}^n} a_l t_1^{l_1}\dots t_n^{l_n}$ where $l=(l_1,\dots,l_n)\in\mathbb{Z}^n$ and $a_l\in A$, with certain restrictions on the set of indices of nonzero coefficients. The goal of this paper is to study the continuous homomorphisms between these types of algebras. First, a description of such homomorphisms is given. Indeed, let $\phi_1,\dots,\phi_n\in\mathcal{L}^n(A)^*$ be a collection of n invertible iterated Laurent series in m variables, with some restrictive conditions. Then we have a well defined continuous homomorphism of $A-$algebras $\phi:\mathcal{L}^n(A)\longrightarrow\mathcal{L}^m(A)$, which assigned to each $f=\sum\limits_{l\in\mathbb{Z}^n} a_l t_1^{l_1}\dots t_n^{l_n}$, $\phi(f)=\sum\limits_{l\in\mathbb{Z}^n} a_l \phi_1^{l_1}\dots\phi_n^{l_n}$. Moreover, all the continuous homomorphisms of $A-$algebras $\phi:\mathcal{L}^n(A)\longrightarrow\mathcal{L}^m(A)$ have this form. Then, a criterion of invertibility for endomorphism is given and an explicit formula for the inverse is provided. Other applications are also stated.

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