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A lower bound for the isoperimetric deficit. (English) Zbl 1368.52004

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Given a plane compact convex set K of area F with boundary curve C of length L, the *isoperimetric deficit* is $\Delta := L^2 - 4\pi F$ and the classical *isoperimetric inequality* states $\Delta \geq 0$, with equality only for discs. As for upper bounds, *A. Hurwitz* [Ann. Sci. Éc. Norm. Supér. (3) 19, 357–408 (1902; JFM 33.0599.02)] proved that $0 \leq \Delta \leq \pi|F_e|$, where F_e is the algebraic area encloded by the *evolute* (i.e. the locus of the centres of curvature) of C.

For lower bounds, during the 1920's, T. Bonnesen proved a series of inequalities of the form $\Delta \geq B$, where B has the following three basic properties: it is non-negative; it can vanish only when C is a circle; B has geometric significance. A *Bonnesen-style inequality* is an inequality as above which satisfies the three basic properties.

In the paper under review, the authors prove a Bonnesen-style inequality; to state this, we need some definitions: the *pedal curve* of C with respect to a fixed point O is the locus of points X so that the line $\overline{OX}$ is perpendicular to the tangent to C passing through X; the *Steiner point* of K is the centre of mass of C with respect to the density function that assigns to each point of C its curvature. Let A be the area enclosed by the pedal curve with respect to the Steiner point of K; then, in Theorem 3.1 the authors prove that $\Delta \geq 3\pi(A - F)$. Moreover, the authors improve the above inequality in special cases, and consider also when the equality holds.

Reviewer: Pietro De Poi (Udine)

MSC:

52A40 Inequalities and extremum problems involving convexity in convex geometry

51M16 Inequalities and extremum problems in real or complex geometry

51M25 Length, area and volume in real or complex geometry

52A10 Convex sets in 2 dimensions (including convex curves)

Keywords:

plane compact convex sets; isoperimetric inequality; isoperimetric deficit; evolute; pedal curve; Steiner point; Bonnesen-style inequality

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