Summary: Let G be a graph and let s be a vertex of G. We consider the structure of the set of all lifts of two edges incident with s that preserve edge-connectivity. Mader proved that two mild hypotheses imply there is at least one pair that lifts, while Frank showed (with the same hypotheses) that there are at least $(\deg(s) - 1)/2$ disjoint pairs that lift. We consider the *lifting graph*: its vertices are the edges incident with s, two being adjacent if they form a liftable pair. We have three main results, the first two with the same hypotheses as for Mader's Theorem. (i) Let F be a subset of the edges incident with s. We show that F is independent in the lifting graph of G if and only if there is a single edge-cut C in G of size at most $r + 1$ containing all the edges in F, where r is the maximum number of edge-disjoint paths from a vertex (not s) in one component of $G - C$ to a vertex (not s) in another component of $G - C$. (ii) In the k-lifting graph, two edges incident with s are adjacent if their lifting leaves the resulting graph with the property that any two vertices different from s are joined by k pairwise edge-disjoint paths. If both $\deg(s)$ and k are even, then the k-lifting graph is a connected complete multipartite graph. In all other cases, there are at most two components. If there are exactly two components, then each component is a complete multipartite graph. If $\deg(s)$ is odd and there are two components, then one component is a single vertex. (iii) Huck proved that if k is odd and G is $(k+1)$-edge-connected, then G is weakly k-linked (that is, for any k pairs $\{x_i, y_i\}$, there are k edge-disjoint paths P_i, with P_i joining x_i and y_i). We use our results to extend a slight weakening of Huck's theorem to some infinite graphs: if k is odd, every $(k + 2)$-edge-connected, locally finite, 1-ended, infinite graph is weakly k-linked.

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