Dowdall, Spencer; **Kapovich, Ilya**; **Taylor, Samuel J.**
Cannon-Thurston maps for hyperbolic free group extensions. (English) Zbl 1361.20030
Isr. J. Math. 216, No. 2, 753-797 (2016).

Let H and G be two hyperbolic groups such that H is a subgroup of G. If the inclusion of H into G induces a map from the Gromov boundary of H to that of G, the map goes under the name of *Cannon-Thurston map*. *J. W. Cannon* and *W. P. Thurston* [Geom. Topol. 11, 1315–1355 (2007; Zbl 1136.57009)] studied the case where G is the fundamental group of a closed hyperbolic 3-manifold fibring over the circle, H a surface group being the fundamental group of the fibre. It was shown by *M. Mitra* [Topology 37, No. 3, 527–538 (1998; Zbl 0907.20038)] that if H is normal in G the Cannon-Thurston map exists and is, moreover, surjective. In this setting, it follows that, if H is torsion-free, then it is a free product of surface groups and free groups.

In this paper, the authors analyse the Cannon-Thurston map in the situation where H is a free group of rank at least 3 and G is a hyperbolic extension of H by a *convex cocompact* subgroup of the outer automorphism group of H, generalising work of *I. Kapovich* and *M. Lustig* on free-by-cyclic hyperbolic extensions [J. Lond. Math. Soc., II. Ser. 91, No. 1, 203–224 (2015; Zbl 1325.20035)]. Note that the condition that G/H is convex cocompact implies in particular that the quotient is a hyperbolic group.

A first main result established in the paper shows that, under the above hypotheses, the fibres of the Cannon-Thurston map have size bounded by twice the rank of H. The authors also prove that a point in the Gromov boundary of G, satisfying the technical condition of being *essential*, is *rational* (*irrational*, resp.) if so is its image in the boundary of G/H, provided the fibre of the Cannon-Thurston map over the point is “sufficiently large”, i.e., it contains at least 3 (2, resp.) points. Recall that a point in the boundary of G is rational if it is the limit point of an infinite order element of G, and irrational otherwise.

In the last part of the paper, the authors consider the map that associates to each point in the boundary of G/H an *ending lamination* of H. Recall that a lamination can be seen as a closed subset (with some extra properties) of the set of pairs of distinct points of the boundary of H. The collection of lamination is thus endowed with the Chabauty topology. The authors exhibit an exemple of an extension G for which the above map is not continuous, answering a question of Mitra. On the other hand, they show that for an extension of a free group by a purely atoroidal, convex cocompact group G/H the following property holds: Let z and z_i, $i \in \mathbb{N}$, be points in the boundary of G/H such that the sequence (z_i) converges to z. Let Λ_z and Λ_i, $i \in \mathbb{N}$, be the corresponding ending laminations. Then if L is the limit of a convergent subsequence of (Λ_i), L is contained in Λ_z and contains the accumulation points of Λ_z.

Reviewer: Luisa Paoluzzi (Marseille)

MSC:

20F67 Hyperbolic groups and nonpositively curved groups
20E22 Extensions, wreath products, and other compositions of groups
20E05 Free nonabelian groups
57M07 Topological methods in group theory
20E36 Automorphisms of infinite groups
20F65 Geometric group theory

Keywords:

Cannon-Thurston map; free groups; hyperbolic groups; hyperbolic group extensions; Gromov boundary; laminations

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References:

[1] M. Bestvina and M. Feighn, \textit{Outer limits}, Preprint 1994; http://andromeda.rutgers.edu/feighn/papers/outer.pdf, 2013.

[2] Bestvina, M.; Feighn, M., Hyperbolicity of the complex of free factors, Adv. Math., 256, 104-155, (2014) · Zbl 1348.20028

[3] Bestvina, M.; Feighn, M.; Handel, M., Laminations,trees,and irreducible automorphisms of free groups, Geom. Funct. Anal., 7, 215-244, (1997) · Zbl 0884.57002

[4] Bestvina, M.; Handel, M., Train tracks and automorphisms of free groups, 1-51, (1992) · Zbl 0757.57004

[5] Bowditch, B. H., The Cannon-Thurston map for punctured-surface groups, Math. Z., 255, 35-76, (2007) · Zbl 1138.57020

[6] Bowditch, B. H., Stacks of hyperbolic spaces and ends of 3-manifolds, 65-138, (2013), Providence, RI · Zbl 1297.57044

[7] Baker, O.; Riley, T. R., Cannon-Thurston maps do not always exist, Forum Math. Sigma, 1, e3, (2013) · Zbl 1276.20054

[8] Bestvina, M.; Reynolds, P., The boundary of the complex of free factors, Duke Math. J., 164, 2213-2251, (2015) · Zbl 1337.20040

[9] Brinkmann, P., Hyperbolic automorphisms of free groups, Geom. Funct. Anal., 10, 1071-1089, (2000) · Zbl 0970.20018

[10] Coulbois, T.; Hilion, A., Botany of irreducible automorphisms of free groups, Pacific J. Math., 256, 291-307, (2012) · Zbl 1259.20031

[11] Coulbois, T.; Hilion, A., Rips induction: index of the dual lamination of an R-tree, Groups Geom. Dyn., 8, 97-134, (2014) · Zbl 1336.20033

[12] T. Coulbois, A. Hilion and M. Lustig, \textit{Which R-trees can be mapped continuously to a current?}, Draft preprint, 2006.

[13] Coulbois, T.; Hilion, A.; Lustig, M., Non-unique ergodicity, observers' topology and the dual algebraic lamination for R-trees, Illinois J. Math., 51, 897-911, (2007) · Zbl 1197.20020

[14] Coulbois, T.; Hilion, A.; Lustig, M., R-trees and laminations for free groups. I. algebraic laminations, J. Lond. Math. Soc., 78, 723-736, (2008) · Zbl 1197.20019

[15] Coulbois, T.; Hilion, A.; Lustig, M., R-trees and laminations for free groups. II. the dual lamination of an R-tree, J. Lond. Math. Soc., 78, 737-754, (2008) · Zbl 1198.20023

[16] Coulbois, T.; Hilion, A.; Reynolds, P., Indecomposable FN-trees and minimal laminations, Groups Geom. Dyn., 9, 567-597, (2015) · Zbl 1342.20028

[17] Cohen, M. M.; Lustig, M., Very small group actions on R-trees and Dehn twist automorphisms, Topology, 34, 575-617, (1995) · Zbl 0844.20018

[18] R. D. Canary, A. Marden and D. Epstein, \textit{Fundamentals of hyperbolic manifolds: Selected expositions}, Vol. 328, Cambridge University Press, 2006. · Zbl 1083.30001

[19] Cannon, J. W.; Thurston, W. P., Group invariant Peano curves, Geom. Topol., 11, 1315-1355, (2007) · Zbl 1136.57009

[20] Culler, M.; Vogtmann, K., Moduli of graphs and automorphisms of free groups, Invent. Math., 84, 91-119, (1986) · Zbl 0589.20022

[21] S. Dowdall and S. J. Taylor, \textit{Hyperbolic extensions of free groups}, Preprint arXiv:1406.2567, 2014. · Zbl 1439.20034

[22] S. Dowdall and S. J. Taylor, \textit{The co-surface graph and the geometry of hyperbolic free group extensions}, Preprint arXiv:1601.00101, 2016. · Zbl 1454.20087

[23] Feighn, M.; Handel, M., The recognition theorem for out(fn), Groups Geom. Dyn., 5, 39-106, (2011) · Zbl 1239.20036

[24] Francaviglia, S.; Martino, A., Metric properties of outer space, Publ. Mat., 55, 433-473, (2011) · Zbl 1268.20042

[25] Gerasimov, V., Floyd maps for relatively hyperbolic groups, Geom. Funct. Anal., 22, 1361-1399, (2012) · Zbl 1276.20050

[26] Guirardel, V., Approximations of stable actions on R-trees, Comment. Math. Helv., 73, 89-121, (1998) · Zbl 0979.20026

[27] Guirardel, V., Actions of finitely generated groups on R-trees, Ann. Inst. Fourier (Grenoble), 58, 159-211, (2008) · Zbl 1187.20020

[28] U. Hamenstädt, \textit{The boundary of the free factor graph and the free splitting graph}, Preprint arXiv:1211.1630, 2012.

[29] U. Hamenstädt and S. Hensel, \textit{Convex cocompact subgroups of Out(Fn)}, Preprint arXiv:1411.2281, 2014.

[30] M. Handel and L. Mosher, \textit{Axes in outer space.}, Vol. 213, Mem. Amer. Math. Soc., 2011. · Zbl 1238.57002

[31] Hatcher, A.; Vogtmann, K., The complex of free factors of a free group, Quart. J. Math., 49, 459-468, (1998) · Zbl 0935.20015

[32] Jeon, W.; Kapovich, I.; Leininger, C.; Ohshika, K., Conical limit points and the Cannon-Thurston map, Conform. Geom. Dyn., 20, 58-80, (2016) · Zbl 1375.20045

[33] Jäger, A.; Lustig, M., Free group automorphisms with many fixed points at infinity, Geometry \& Topology Monographs, 14, 321-333, (2008) · Zbl 1140.20027

[34] Kapovich, I.; Lustig, M., Geometric intersection number and analogues of the curve complex for free groups, Geom. Topol., 13, 1805-1833, (2009) · Zbl 1194.20046

[35] Kapovich, I.; Lustig, M., Ping-pong and outer space, J. Topol. Anal., 2, 173-201, (2010) · Zbl 1211.20027

[36] Kapovich, I.; Lustig, M., Stabilizers of R-trees with free isometric actions of FN, J. Group Theory, 14, 673-694, (2011) · Zbl 1262.20031

[37] Kapovich, I.; Lustig, M., Invariant laminations for irreducible automorphisms of free groups, Q. J. Math., 65, 1241-1275, (2014) · Zbl 1348.20035

[38] Kapovich, I.; Lustig, M., Cannon-Thurston fibers for iwip automorphisms of FN, J. Lond. Math. Soc., 91, 203-224, (2015) · Zbl 1325.20035

[39] Klarreich, E., Semiconjugacies between Kleinian group actions on the Riemann sphere, Amer. J. Math., 121, 1031-1078, (1999)

· Zbl 1011.30035

[40] Levitt, G., Automorphisms of hyperbolic groups and graphs of groups, Geom. Dedicata, 114, 49-70, (2005) · Zbl 1107.20030

[41] Leininger, C.; Long, D. D.; Reid, A. W., Commensurators of finitely generated nonfree Kleinian groups, Algebr. Geom. Topol., 11, 605-624, (2011) · Zbl 1237.20044

[42] Leininger, C. J.; Mj, M.; Schleimer, S., The universal Cannon-Thurston map and the boundary of the curve complex, Comment. Math. Helv., 86, 769-816, (2011) · Zbl 1248.57003

[43] B. Mann, \textit{Some hyperbolic Out(FN)-graphs and nonunique ergodicity of very small FN-trees}, ProQuest LLC, Ann Arbor, MI, 2014, Thesis (Ph.D.)-The University of Utah.

[44] McMullen, C. T., Local connectivity, Kleinian groups and geodesics on the blowup of the torus, Invent. Math., 146, 35-91, (2001) · Zbl 1061.37025

[45] Mitra, M., Ending laminations for hyperbolic group extensions, Geom. Funct. Anal., 7, 379-402, (1997) · Zbl 0880.57001

[46] Mitra, M., Cannon-Thurston maps for hyperbolic group extensions, Topology, 37, 527-538, (1998) · Zbl 0907.20038

[47] Mitra, M., Cannon-Thurston maps for trees of hyperbolic metric spaces, J. Differential Geom., 48, 135-164, (1998) · Zbl 0906.20023

[48] Mitra, M., Coarse extrinsic geometry: a survey, 341-364, (1998) · Zbl 0914.20034

[49] Miyachi, H., Moduli of continuity of Cannon-Thurston maps, 121-149, (2006), Cambridge · Zbl 1098.30032

[50] Mj, M., Cannon-Thurston maps for surface groups, Ann. of Math., 179, 1-80, (2014) · Zbl 1301.57013

[51] Mj, M., Ending laminations and Cannon-Thurston maps, Geom. Funct. Anal., 24, 297-321, (2014) · Zbl 1297.57040

[52] Mj, M.; Pal, A., Relative hyperbolicity, trees of spaces and Cannon-Thurston maps, Geom. Dedicata, 151, 59-78, (2011) · Zbl 1222.57013

[53] B. Mann and P. Reynolds, In preparation. · Zbl 0910.57002

[54] M. Mj and K. Rafi, \textit{Algebraic ending laminations and quasiconvexity}, Preprint arXiv:1506.08036v2, 2015. · Zbl 06867651

[55] H. Namazi, A. Pettet and P. Reynolds, \textit{Ergodic decompositions for folding and unfolding paths in outer space}, Preprint arXiv:1410.8870, 2014.

[56] Paulin, F., The Gromov topology on R-trees, Topology Appl., 32, 197-221, (1989) · Zbl 0675.20033

[57] Pfaff, C., Out(F3) index realization, Math. Proc. Cambridge Philos. Soc., 159, 445-458, (2015) · Zbl 1377.20021

[58] P. Reynolds, \textit{Reducing systems for very small trees}, Preprint arXiv:1211.3378, 2012.

[59] Rips, E.; Sela, Z., Cyclic splittings of finitely presented groups and the canonical JSJ decomposition, Ann. of Math. (2), 146, 53-109, (1997) · Zbl 0910.57002

[60] Taylor, S. J.; Tiozzo, G., Random extensions of free groups and surface groups are hyperbolic, 294-310, (2016) · Zbl 1368.20060