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Central theorems for cohomologies of certain solvable groups. (English) Zbl 1401.20035

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Summary: We show that the group cohomology of torsion-free virtually polycyclic groups and the continuous cohomology of simply connected solvable Lie groups can be computed by the rational cohomology of algebraic groups. Our results are generalizations of certain results on the cohomology of solvmanifolds and infra-solvmanifolds. Moreover as an application of our results, we give a new proof of the surprising cohomology vanishing theorem given by *K. Dekimpe* and *P. Igodt* [Invent. Math. 129, No. 1, 121–140 (1997; Zbl 0867.20031)].

MSC:

20F16 Solvable groups, supersolvable groups
20G10 Cohomology theory for linear algebraic groups
20J06 Cohomology of groups
22E41 Continuous cohomology of Lie groups
22E25 Nilpotent and solvable Lie groups
17B56 Cohomology of Lie (super)algebras
57T15 Homology and cohomology of homogeneous spaces of Lie groups

Keywords:

group cohomology of torsion-free virtually polycyclic group; continuous cohomology of simply connected solvable Lie group; rational cohomology of algebraic group; de Rham cohomology of solvmanifold

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