Wu, C. T.; **Hu, Wei**; **Liu, G. R.**

Bubble-enhanced smoothed finite element formulation: a variational multi-scale approach for volume-constrained problems in two-dimensional linear elasticity. (English)

Zbl 1352.74451
Int. J. Numer. Methods Eng. 100, No. 5, 374-398 (2014).

Summary: This paper presents a bubble-enhanced smoothed finite element formulation for the analysis of volume-constrained problems in two-dimensional linear elasticity. The new formulation is derived based on the variational multi-scale approach in which unequal order displacement-pressure pairs are used for the mixed finite element approximation and hierarchical bubble function is selected for the fine-scale displacement approximation. An area-weighted averaging scheme is employed for the two-scale smoothed strain calculation under the framework of edge-based smoothed FEM. The smoothed fine-scale solution is shown to naturally contain the stress field jump of the smoothed coarse-scale solution across the boundary of edge-based smoothing domain and thus provides the possibility to stabilize the global solution for volume-constrained problems. A global monolithic solution strategy is employed, and the fine-scale solution is solved without the consideration of approximating the strong form of the fine-scale equation. Several numerical examples are analyzed to demonstrate the accuracy of the present formulation.

MSC:

74S05 Finite element methods applied to problems in solid mechanics
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
74B05 Classical linear elasticity

Keywords:

multi-scale; smoothed finite element; bubble function; edge-based smoothing

Full Text: DOI

References:

[1] Liu, A smoothed finite element method for mechanics problems, Computational Mechanics 39 pp 859– (2007) · Zbl 1169.74047 · doi:10.1007/s00466-006-0075-4

[2] Liu, Smoothed Finite Element Methods (2010) · doi:10.1201/EBK1439820278

[3] Chen, A stabilized conforming nodal integration for Galerkin mesh-free methods, International Journal for Numerical Methods in Engineering 50 pp 435– (2001) · Zbl 1011.74081 · doi:10.1002/1097-0207(20010120)50:2<435::AID-NME32>3.0.CO;2-A

[4] Wu, A two-level mesh repartitioning scheme for the displacement-based lower-order finite element methods in volumetric locking-free analyses, Computational Mechanics 50 pp 1– (2012) · Zbl 1312.74048 · doi:10.1007/s00466-011-0665-7

[5] Liu, A node-based smoothed finite element method (NS-FEM) for upper bound solutions to solid mechanics problems, Computers and Structures 87 pp 14– (2009) · doi:10.1016/j.compstruc.2008.09.003

[6] Liu, An edge-based smoothed finite element method (ES-FEM) for static, free and forced vibration analyses in solids, Journal of Sound and Vibration 320 pp 1100– (2009) · doi:10.1016/j.jsv.2008.08.027

[7] Nguyen-Xuan, A smoothed finite element method for plate analysis, Computer Methods in Applied Mechanics and Engineering 197 pp 1184– (2008) · Zbl 1159.74434 · doi:10.1016/j.cma.2007.10.008

[8] Wu, An enhanced cell-based smoothed finite element method for the analysis of Reissner-Mindlin plate bending problems involving distorted mesh, International Journal for Numerical Methods in Engineering 95 pp 288– (2013) · Zbl 1352.74452 · doi:10.1002/nme.4506

[9] Nguyen, Selective smoothed finite element method, Tsinghua Science and Technology 12 (5) pp 497– (2007) · doi:10.1016/S1007-0214(07)70125-6

[10] Nguyen-Xuan, Smoothed finite elements: convergence, accuracy and properties, International Journal for Numerical Methods in Engineering 74 pp 175– (2008) · Zbl 1159.74435 · doi:10.1002/nme.2146

[11] Brezzi, Springer Series in Computational Mathematics, in: Mixed and Hybrid Finite Element Methods (1991) · Zbl 0788.73002 · doi:10.1007/978-1-4612-3172-1

[12] Zienkiewicz, The Finite Element Method, 3. ed. (1987) · Zbl 0623.65120

[13] Hughes, The Finite Element Method (2000) · Zbl 1191.74002

[14] Belytschko, Nonlinear Finite Elements for Continua and Structures (2001)

[15] Liu, Finite element stabilization matrices - a unification approach, Computer Methods in Applied Mechanics and Engineering 53 pp 13– (1986)

[16] Areias, Stabilized four-node tetrahedron with nonlocal pressure for modeling hyperelastic materials, International Journal for Numerical Methods in Engineering 76 pp 1185– (2008) · Zbl 1195.74157 · doi:10.1002/nme.2357

[17] Hughes, A new finite element formulation for fluid dynamics, V. Circumventing the Babuška-Brezzi condition: a stable Petrov-Galerkin formulation of the Stokes problem accommodating equal order interpolation, Computer Methods in Applied Mechanics and Engineering 59 pp 85– (1986) · Zbl 0622.76077

[18] Simo, A class of mixed assumed strain methods and the method of incompatible modes, International Journal for Numerical Methods in Engineering 29 pp 1595– (1990) · Zbl 0724.73222 · doi:10.1002/nme.1620290802

[19] Arnold, Unified analysis of discontinuous Galerkin methods for elliptic problems, SIAM Journal on Numerical Analysis 39 pp 1749– (2002) · Zbl 1008.65080 · doi:10.1137/S0036142901384162

[20] Hansbo, Discontinuous Galerkin methods for incompressible and nearly incompressible elasticity by Nitsche's method, Computer Methods in Applied Mechanics and Engineering 191 pp 1895– (2002) · Zbl 1098.74693 · doi:10.1016/S0045-7825(01)00358-9

[21] Bonet, A simple average nodal pressure tetrahedral element for incompressible and nearly incompressible dynamic explicit applications, Communications in Numerical Methods in Engineering 14 pp 437– (1998) · Zbl 0906.73060 · doi:10.1002/(SICI)1099-0887(199805)14:5<437::AID-CNM162>3.0.CO;2-W

[22] Hauret, Diamond elements: a finite element/discrete-mechanics approximation scheme with guaranteed optimal convergence in incompressible elasticity, International Journal for Numerical Methods in Engineering 72 pp 253– (2007) · Zbl 1194.74406 · doi:10.1002/nme.1992

[23] Wriggers, Brick elements for finite deformations based on macro-concepts and on inhomogeneous enhancement, Computer Methods in Applied Sciences 14 pp 33– (2009) · Zbl 1185.74095

[24] Stenberg, Error analysis of some finite element methods for the Stokes problem, Mathematics of Computation 54 pp 495– (1990) · Zbl 0702.65095 · doi:10.1090/S0025-5718-1990-1010601-X

[25] Vidal, Locking in the incompressible limit: pseudo-divergence-free element free Galerkin, Communications in Numerical Methods in Engineering 19 pp 725– (2003) · Zbl 1112.74545 · doi:10.1002/cnm.631

[26] Belytschko, Element-free Galerkin methods, International Journal for Numerical Methods in Engineering 37 pp 229– (1994) · Zbl 0796.73077 · doi:10.1002/nme.1620370205

[27] Chen, An improved reproducing kernel particle method for nearly incompressible finite elasticity, Computer Methods in Applied Mechanics and Engineering 181 pp 117– (2000) · Zbl 0973.74088 · doi:10.1016/S0045-7825(99)00067-5

[28] Liu, Reproducing kernel particle methods, International Journal for Numerical Methods in Fluids 20 pp 1081– (1995) · Zbl 0881.76072 · doi:10.1002/fld.1650200824

[29] Auricchio, A fully locking-free isogeometric approach for plane linear elasticity problems: a stream function formulation, Computer Methods in Applied Mechanics and Engineering 197 pp 160– (2007) · Zbl 1169.74643 · doi:10.1016/j.cma.2007.07.005

[30] Ortiz, Maximum-entropy meshfree method for compressible and near-incompressible elasticity, Computer Methods in Applied Mechanics and Engineering 199 pp 1859– (2010) · Zbl 1231.74491 · doi:10.1016/j.cma.2010.02.013

[31] Wu, Meshfree-enriched simplex elements with strain smoothing for the finite element analysis of compressible and nearly incompressible solids, Computer Methods in Applied Mechanics and Engineering 200 pp 2991– (2011) · Zbl 1230.74201 · doi:10.1016/j.cma.2011.06.013

[32] Wu, A meshfree-enriched finite element method for compressible and near-incompressible elasticity, International Journal for Numerical Methods in Engineering 90 pp 882– (2012) · Zbl 1242.74174 · doi:10.1002/nme.3349

[33] Babuška, The finite element method with Lagrangian multipliers, Numerische Mathematik 20 pp 179– (1973) · Zbl 0258.65108 · doi:10.1007/BF01436561

[34] Hughes, A new finite element formulation for computational fluid dynamics: V. Circumventing the Babuška-Brezzi condition: a stable Petro-Galerkin formulation of the Stokes problem accommodating equal-order interpolations, Computer Methods in Applied Mechanics and Engineering 59 pp 85– (1986) · Zbl 0622.76077 · doi:10.1016/0045-7825(86)90025-3

[35] Hughes, Finite Elements in Fluids pp 46– (1982)

[36] Hughes, A new finite element formulation for computational fluid dynamics: VIII. The galerkin/least-squares method for advective-diffusive equations, Computer Methods in Applied Mechanics and Engineering 73 pp 173– (1989) · Zbl 0697.76100 · doi:10.1016/0045-7825(89)90111-4

[37] Brezzi, A relationship between stabilized finite element methods and the Galerkin method with bubble functions, Computer Methods in Applied Mechanics and Engineering 96 pp 117– (1992) · Zbl 0756.76044 · doi:10.1016/0045-7825(92)90102-P

[38] Hughes, Multiscale phenomena: Green's functions, the Dirichlet-to-Neumann formulation, subgrid scale methods, bubbles and the origins of stabilized methods, Computer Methods in Applied Mechanics and Engineering 127 pp 387– (1995) · Zbl 0866.76044 · doi:10.1016/0045-7825(95)00844-9

[39] Hughes, Variational multiscale analysis: the fine-scale Green's function, projection, optimization, localization, and the stabilized methods, SIAM Journal on Numerical Analysis 45 pp 539– (2007) · Zbl 1152.65111 · doi:10.1137/050645646

[40] Franca, Refining the submesh strategy in the two-level finite element method: application to the advection-diffusion equation, International Journal for Numerical Methods in Engineering 39 pp 161– (2002) · Zbl 1016.76047 · doi:10.1002/fld.219

[41] Araya, Stabilized finite element methods based on multiscale enrichement for the Stokes problem, SIAM Journal on Numerical Analysis 44 pp 322– (2006) · Zbl 1121.65116 · doi:10.1137/050623176

[42] Franca, Towards multiscale functions: enriching finite element spaces with local but not bubble-like functions, Computer Methods in Applied Mechanics and Engineering 194 pp 3006– (2005) · Zbl 1091.76034 · doi:10.1016/j.cma.2004.07.029

[43] Farhat, The discontinuous enrichment method for multiscale analysis, Computer Methods in Applied Mechanics and Engineering 192 pp 3195– (2003) · Zbl 1054.76048 · doi:10.1016/S0045-7825(03)00344-X

[44] Baiges, A variational multiscale method with subscales on the element boundaries for the Helmholtz equation, International Journal for Numerical Methods in Engineering 93 pp 664– (2012) · Zbl 1352.65467 · doi:10.1002/nme.4406

[45] Wu, Multi-scale finite element analysis of acoustic waves using global residual-free meshfree enrichments, Interaction and Multiscale Mechanics 6 (2) pp 83– (2013) · doi:10.12989/imm.2013.6.2.083

[46] Hauke, variational subgrid scale formulations for the advection-diffusion-reaction equation, Computer Methods in Applied Mechanics and Engineering 190 pp 6847– (2001) · Zbl 0996.76074 · doi:10.1016/S0045-7825(01)00262-6

[47] Masud, A multiscale finite element method for the incompressible Navior-Stokes equations, Computer Methods in Applied Mechanics and Engineering 195 pp 1750– (2006) · Zbl 1178.76233 · doi:10.1016/j.cma.2005.05.048

[48] Liu, A variational multiscale stabilized finite element method for the Stokes flow problem, Finite Elements in Analysis and Design 42 pp 580– (2006) · doi:10.1016/j.finel.2005.11.006

[49] Oberai, A multiscale finite element method for the Helmholtz equation, Computer Methods in Applied Mechanics and Engineering 154 pp 281– (1998) · Zbl 0937.65119 · doi:10.1016/S0045-7825(97)00130-8

[50] Harari, Multiscale finite elements for acoustics: continuous, discontinuous, and stabilized methods, International Journal for Numerical Methods in Engineering 6 pp 511– (2008)

[51] Masud, A stabilized mixed finite element method for nearly incompressible elasticity, Journal of Applied Mechanics 72 pp 711– (2005) · Zbl 1111.74548 · doi:10.1115/1.1985433

[52] Masud, A variational multiscale method for computational inelasticity: application to superelasticity in shape memory alloys, Computer Methods in Applied Mechanics and Engineering 195 pp 4512– (2006) · Zbl 1123.74046 · doi:10.1016/j.cma.2005.09.014

[53] Nakshatrala, On finite element formulations for nearly incompressible linear elasticity, Computational Mechanics 41 pp 547– (2008) · Zbl 1162.74472 · doi:10.1007/s00466-007-0212-8

[54] Bochev, Stabilization of low-order mixed finite elements for the Stokes equations, SIAM Journal on Numerical Analysis 44 pp 82– (2006) · Zbl 1145.76015 · doi:10.1137/S0036142905444482

[55] Franca, Approximation of the Stokes problem by residual-free macro bubbles, East-West Journal of Mathematics 4 pp 265– (1996) · Zbl 0869.76038

[56] Whiting, A stabilized finite element method for the incompressible Navier-Stokes equations using a hierarchical basis, International Journal for Numerical Methods in Fluids 35 pp 93– (2001) · Zbl 0990.76048 · doi:10.1002/1097-0363(20010115)35:1<93::AID-FLD85>3.0.CO;2-G

[57] Timoshenko, Theory of Elasticity (1970)

[58] Lovadina, On the enhanced strain technique for elasticity problems, Computers and Structures 81 pp 777– (2003) · doi:10.1016/S0045-7949(02)00412-1