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**Normal forms and normal theories in conditional rewriting.** (English) Zbl 1356.68124  
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Summary: We present several new concepts and results on conditional term rewriting within the general framework of order-sorted rewrite theories (OSRTs), which support types, subtypes and rewriting modulo axioms, and contains the more restricted framework of conditional term rewriting systems (CTRSs) as a special case. The concepts shed light on several subtle issues about conditional rewriting and conditional termination. We point out that the notions of *irreducible* term and of *normal form*, which coincide for unconditional rewriting, have been conflated for conditional rewriting but are in fact *totally different* notions. Normal form is a *stronger* concept. We call any rewrite theory where all irreducible terms are normal forms a *normal* theory. We argue that normality is essential to have good executability and computability properties. Therefore we call all other theories *abnormal*, freaks of nature to be avoided. The distinction between irreducible terms and normal forms helps in clarifying various notions of strong and weak termination. We show that abnormal theories can be terminating in various, equally abnormal ways; and argue that any computationally meaningful notion of strong or weak conditional termination should be a property of normal theories. In particular we define the notion of a *weakly operationally terminating* (or *weakly normalizing*) OSRT, discuss several evaluation mechanisms to compute normal forms in such theories, and investigate general conditions under which the rewriting-based operational semantics and the initial algebra semantics of a confluent, weakly normalizing OSRT coincide thanks to a notion of *canonical term algebra*. Finally, we investigate appropriate conditions and proof methods to ensure that a rewrite theory is normal; and characterize the stronger property of a rewrite theory being operationally terminating in terms of a natural generalization of the notion of quasi-decreasing order.

**MSC:**

68Q42 Grammars and rewriting systems

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**Keywords:**

conditional term rewriting; normal forms; normal theory; operational termination; rewriting logic; Maude

**Software:**

CafeOBJ; Maude; OBJ3

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